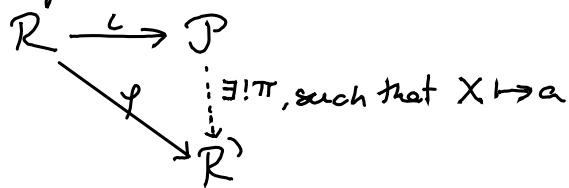
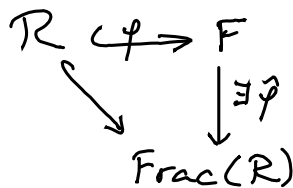


- **Ring** R :
 - $(R, +)$ abelian
 - $(R, \cdot)$ monoid
 - distributive law
- $\frac{R/I}{J/I} \cong \frac{R}{J}$
- $\ker(\text{eval}_a) = \langle X - a \rangle$, $\frac{P}{\ker(\text{eval}_a)} \cong R$
- e, e' complementary idempotents: $R \cong Re \times Re'$, $Re \cong \frac{R}{Re'}$
- **multiplicative** set S : $1 \in S$ $\wedge$ $S \cdot S \subseteq S$
- $P \trianglelefteq R$ is **prime** if $R - P$ is multiplicative
 $(\Leftrightarrow (ab \in P \Rightarrow a \in P \vee b \in P))$
- **integral domain** is a Ring with no zero divisors
 $(\langle 0 \rangle \text{ is prime})$
- UL for polynomials



- R is integral domain $\Leftrightarrow R$ is subring of a field
- R is integral domain $\Rightarrow$ we can construct $\text{frac}(R)$
- UL for $\text{frac}(R)$



- J prime $\Rightarrow \varphi^*(J)$ prime
- I prime, φ surj $\Rightarrow \varphi_*(I)$ prime
- $P \subseteq R$ is prime $\Leftrightarrow R/P$ is integral domain
- R is a field $\Leftrightarrow \langle 0 \rangle$ is max ideal
- $M \subseteq R$ max ideal $\Leftrightarrow R/M$ is a field
- M max ideal $\Rightarrow M$ prime ideal
- every proper ideal is contained in max ideal
- $x \in R^\times \Leftrightarrow x \notin \text{max ideal}$
- a is irreducible $\Leftrightarrow a$ has no proper divisors $\Leftrightarrow \langle a \rangle$ is max among principal ideals
- p is prime element $\Leftrightarrow p \notin R^\times \wedge (p|ab \Rightarrow p|a \vee p|b)$
 $\Leftrightarrow \langle p \rangle$ is prime ideal
- **unique factorisation domain (UFD)**
 - 1) $\forall a \in R - \{0\}$ is a product of irreducible elem. or units
 - 2) factorisation is unique up to order and units
- R is int domain $\Rightarrow (R \text{ is UFD} \Leftrightarrow$
 - 1) every chain of principle ideals $\langle a_1 \rangle \subseteq \langle a_2 \rangle \subseteq \dots$ stabilizes
 - 2) p irreducible $\Rightarrow p$ prime
- **principal ideal domain (PID)** if every ideal is principal

- R PID $\Rightarrow$
 - p irreducible $\Rightarrow p$ prime
 - $I \neq \langle 0 \rangle \triangleleft R \Rightarrow I$ is max
 - R is UFD
- Jacobson radical $\text{rad}(R) = \bigcap_{M \triangleleft R} M$