

Naj bo $x, y \in A^*$. Pokaži da je enekost $x y = y x$
 $x y = y x \Leftrightarrow \exists z \in A^*. x = z^n, y = z^m$

$$(\Leftarrow) \quad z^n z^m = z^{n+m} = z^{m+n} = z^m z^n$$

$(\Rightarrow)$

$$x_1 \dots x_n y_1 \dots y_m = y_1 \dots y_m x_1 \dots x_n$$

BŠZS $n \geq m$

$$x_1 = y_1, \dots, x_m = y_m = x_n$$

$$x_{m-1} = x_{m-1} = x_{n-1}$$

$\vdots$

$$y_1 = x_1 = x_{n-m}$$

y je prefix x -a

$$x = yw$$

$$yw y = y y w \quad |w| = n - m$$

$$w y = y w$$

Indukcija na $n+m$

$$n+m=1 \Rightarrow n=1 \quad m=0$$

$$y = \lambda = z^0 \quad x = z$$

Recimo da velja to za manje besed od $n+m$

$$w y = y w$$

$$1) y = \lambda \Rightarrow |w| = |x| \Rightarrow w = x$$

$$y = x^0 \quad \checkmark$$

$$2) y \neq \lambda \Rightarrow |w| < |x| \Rightarrow |y+w| < n+m \Rightarrow$$

$$w = z^{n-m} y = z^m \leadsto \exists x = z^n \quad y = z^m$$

$$12) \quad x, y \in A^*$$

Pravimo da sta konjugirani; $(x \sim y)$ če
 $\exists u, v \in A^*. x = uv, y = vu.$

$$x \sim y \Leftrightarrow \exists z. xz = zy$$

$$(\Rightarrow) \quad x \sim y \quad (x = uv \quad y = vu)$$

$$uvu = uy$$

$$\parallel$$

$$xu \quad \text{torej } \exists u \in A^*. xu = uy.$$

$$(\Leftarrow) \quad \text{Recimo da } xz = zy$$

$$\text{Potem } |x| = |y|$$

$$1) \quad |z| \leq |x|$$

$$\Rightarrow z \text{ je prefix } x$$

$$x = z \cdot w \leadsto$$

$$\Rightarrow z \text{ je postfix } y$$

$$y = rz$$

$$\cancel{yz}$$

$$zwz = zr z$$

$$\Rightarrow w = r$$

$$\Rightarrow x = zw \quad y = wz$$

$$2) \quad |z| > |x|$$

$$\Rightarrow x \text{ je prefix } z \quad z = xw$$

$$\Rightarrow y \text{ je postfix } z \quad z = ry$$

$$xry = xwy \leadsto r = w$$

$$xz = xxw = ryy$$

⋮

$$xz = xxx \dots xw_n = r_n y \dots y = zy$$

Potem je w_n

⋮

$$x \sim y \quad y \sim z \quad z \sim w$$

$$\exists a \quad xa = ay$$

$$\exists b \quad yb = bz$$

$$\exists c \quad zc = cw$$

$$xab = ayb = bz$$

$$x \underbrace{abc} = b \underbrace{zc} = \underbrace{bc} w$$

$$2.1) \quad a \in A \quad L \subseteq A^*$$

$$\alpha^{-1} L^* = \alpha^{-1} \bigcup_{n=0}^{\infty} L^n = \alpha^{-1}(\{\lambda\} \cup L \bigcup_{n=2}^{\infty} L^n) =$$

$$= \emptyset \cup \alpha^{-1} L \cup \alpha^{-1} \bigcup_{n=2}^{\infty} L^n =$$

$$= \alpha^{-1} L \cup \alpha^{-1} \{u \mid u \in \bigcup_{n=2}^{\infty} L^n\} =$$

$$= \alpha^{-1} L \cup \alpha^{-1} \{u \in \bigcup_{n=1}^{\infty} L^n\} =$$

$$= \alpha^{-1} L \cup (\emptyset \cup \alpha^{-1} \bigcup_{n=1}^{\infty} L^n) = (\alpha^{-1} L) \cup L^*$$

2.2)

$$a) \quad \underline{X^{-1}(L_1 + L_2)} = \underline{X^{-1}L_1} + \underline{X^{-1}L_2}$$

$$\bigcup_{a \in X} \alpha^{-1}(L_1 + L_2) = \bigcup_{a \in X} (\alpha^{-1}L_1 \cup \alpha^{-1}L_2) =$$

$$= \bigcup_{a \in X} \alpha^{-1}L_1 \cup \bigcup_{a \in X} \alpha^{-1}L_2 = X^{-1}L_1 + X^{-1}L_2$$

$$b) (Xy)^{-1}L = y^{-1}(X^{-1}(L))$$

$$\bigcup_{\substack{a \in X \\ b \in y}} (ab)^{-1}L = \bigcup_{\substack{a \in X \\ b \in y}} b^{-1}(a^{-1}(L)) = \bigcup_{a \in A} y^{-1}(a^{-1}L) = y^{-1}(X^{-1}L)$$

$$c) (X^*)^{-1}L = (X^*)^{-1}(X^{-1}L) + L$$

~~$$\begin{aligned} \left(\bigcup_{n=0}^{\infty} X^n \right)^{-1} L &= \bigcup_{a \in X^*} a^{-1}L = \bigcup_{a \in \sum_{n=0}^{\infty} X^n} a^{-1}L = \\ &= \bigcup_{a \in \sum_{n=1}^{\infty} X^n} a^{-1}(X^{-1}L) = \bigcup_{a \in X^*} (Xa)^{-1}L \end{aligned}$$~~

$$(X^*)^{-1}L = \sum_{a \in X^*} a^{-1}L = L + \sum_{a \in X^+} a^{-1}L =$$

$$= L + \sum_{a \in X} \sum_{v \in X^*} (av)^{-1}L = L + \sum_{v \in X^*} v^{-1}(X^{-1}L) =$$

$$= (X^*)^{-1}(X^{-1}L) + L$$

$$\varphi: A^* \longrightarrow B^* \quad L \in A^* \text{ racionalen}$$

$$\underline{\varphi(L) \in B \text{ je racionalen}}$$

$$\varphi(\emptyset) = \emptyset$$

$$\varphi(a) \in B^*$$

$$1) \varphi(L_1 + L_2) = \varphi(L_1) + \varphi(L_2)$$

$$2) \varphi(L_1 L_2) = \varphi(L_1) \varphi(L_2)$$

$$3) \varphi(L^*) = \varphi(L)^*$$

$$1) \varphi_*\{\varphi(u); u \in L_1 \cup L_2\} = \{\varphi(u); u \in L_1 \cup L_2\} = \varphi_*(L_1) \cup \varphi_*(L_2)$$

$$2) \text{ Podobno}$$

$$3) \varphi(\{u; u \in \bigcup_{n=0}^{\infty} L^n\}) = \varphi_*(L)^*$$

5)

a) $0^* a b 1^*$

$$0 = \emptyset$$

$$0^* = 1 + 0 + 0^2 + \dots = \{\lambda\}$$

$$1 = \{\lambda\}$$

$$1 a b 1 = a b$$

$$b) ((a b b)^*)^* = \left(\sum_{n=0}^{\infty} (a b b)^n \right)^* =$$

$$= \sum_{k=0}^{\infty} \left(\sum_{n=0}^{\infty} (a b b)^n \right)^k =$$

$$= \sum_{k=0}^{\infty} \sum_{n=0}^{\infty} \binom{n+k}{k} (a b b)^n = \sum_{n=0}^{\infty} (a b b)^n = (a b b)^*$$

c) $a^* b + a^* b a^* = a^* b (1 + a^*) = a^* b a^*$

$$a^* b + a^* b a^* = a^* b (1 + a^*) = a^* b a^*$$

$$(a^* b)^* a^* =$$

$$\left(\sum_{n=0}^{\infty} a^* b \right) a^* = \sum_{n=0}^{\infty} a^* b a^* = \sum_{n=0}^{\infty} \sum_{k=0}^{\infty} a^* b a^k =$$

$$= \sum_{n=0}^{\infty} (\lambda \cdot b + a^+ b) + (b \cdot \lambda + b \cdot a^+)$$

$$(a+b)^* = \sum_{n=0}^{\infty} a+b = \sum_{n=0}^{\infty} a + \sum_{n=0}^{\infty} b = a^* + b^*$$

$$A = \{a, b\}$$

jeziki niso prepoznavni:

$$a) \{u \in A^* ; |u_a| = |u_b|\}$$

$$u = a^m b^m$$

$$x = a^m$$

$$y = a^l$$

;

$$b) \{a^l b^m ; l, m \geq 0 \text{ } l = 2m\}$$

$$a^{2m} b^m$$

$$n = 2m$$

$$\Rightarrow |xy| \leq 2m \Rightarrow x = a^k \quad y = a^l$$

$$z = a^{2m-k-l} b^m$$

$$xy^r z = a^{k+rp+2m-k-l} b^m =$$

$$a^{2m+p(r-1)} b^m$$

$p \geq 1$ za $r \geq 2$ ta beseda ni v jeziku

$$c) \{u \in A^*; u = \tilde{u}\}$$

abababab

aabbaabbaa

aaaaabaaaa

Naj bo beseda dolga $2n+1$:

$$a^n b a^n$$

$$x = a^k$$

$$y = a^l$$

$$z = a^{n-k-l} b a^n$$

$$xy^m z = a^{n+ml-l} b a^n$$

$$l \geq 1 \quad m \geq 2 \Rightarrow \text{ni palindroma}$$

c)

fiksiramo n $n >$

$a_1 \dots a_p$

$$|X| = k \quad k+l < n$$

$$|Y| = l \quad k+ml+p-k-l =$$

$$|Z| = p-k-l$$

$$p+l(m-1)$$

kde j to ni pravištevilo?

$$m-1 = p$$

$$\Rightarrow p \mid p+l \cdot p$$

$$|u|_a = |u|_b \quad |v|_a \geq |v|_b \quad v \text{ prefix of } u$$

Dyck word

③ $\{a^p b^q \mid p \neq q\}$

Recimo da je prepoznaven

Potem je komplement prepoznaven

$$M = \{a^p b^q \mid p, q \geq 0\} = a^* b^*$$

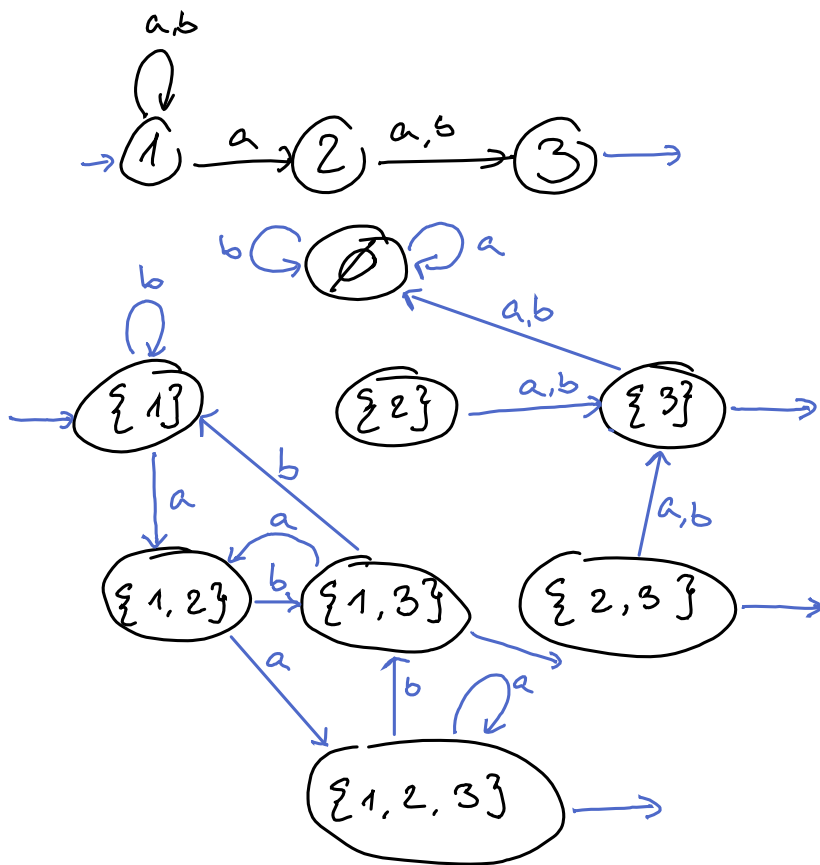
$$L^c \cap M = \{a^p b^q \mid p = q\}$$

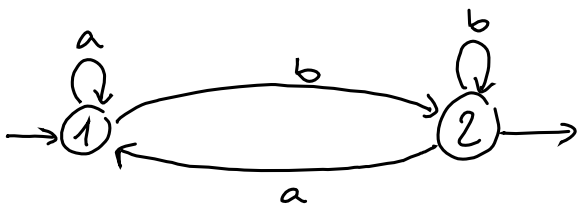


to pa ni prepoznavno



s konstrukcijo



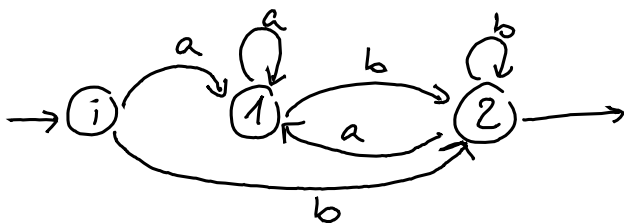


a) Določite racionalni izraz

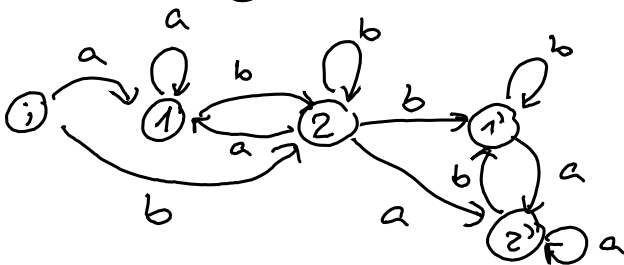
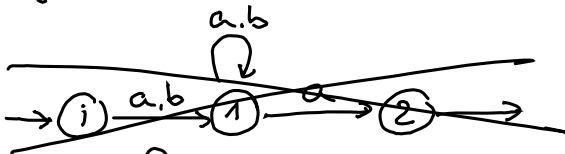
$$a^* b b^* + a^* b b^* a a b b^* + \dots$$

$$= (a^* b b^* a)^* \cdot a^* b b^*$$

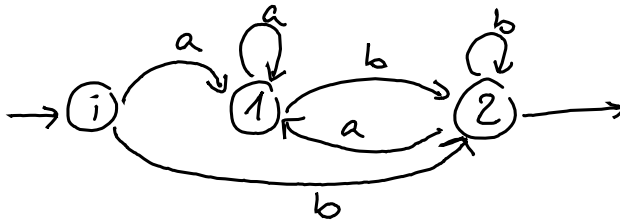
b)



c) $L_2 = (a+b)^* a$



d) $L_1 + L_2$



At