

Fourierove vrste

$$FV(f) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(nx) + b_n \sin(nx)$$

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) dx$$

$$\begin{array}{l} f \text{ sode} \Rightarrow b_n = 0 \\ f \text{ lica} \Rightarrow a_n = 0 \end{array}$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos(nx) dx$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin(nx) dx$$

Parsevalov enekast

$$\frac{1}{\pi} \int_{-\pi}^{\pi} |f(x)|^2 dx = \frac{a_0^2}{2} + \sum_{n=1}^{\infty} (a_n^2 + b_n^2)$$

$$\int x \cos(nx) dx = \frac{x \sin(nx)}{n} + \frac{\cos(nx)}{n^2} + C$$

$$\int x \sin(nx) dx = -\frac{x \cos(nx)}{n} + \frac{\sin(nx)}{n^2} + C$$

$$\int x^2 \cos(nx) dx = \frac{x^2}{n} \sin(nx) + \frac{2x}{n^2} \cos(nx) - \frac{2}{n^3} \sin(nx)$$

$$\int x^2 \sin(nx) dx = -\frac{x^2}{n} \cos x + \frac{2x}{n^2} \sin(nx) + \frac{2}{n^3} \cos(nx)$$

$$\int \cos(nx) dx + i \int \sin(nx) dx = \int e^{inx} dx$$

kathe zedve

$$\sin x \sin y = -\frac{1}{2} (\cos(x+y) - \cos(x-y))$$

$$\cos x \cos y = \frac{1}{2} (\cos(x+y) + \cos(x-y))$$

$$\sin x \cos y = \frac{1}{2} (\cos(x-y) - \cos(x+y))$$

$$\sin x + \sin y = 2 \sin \frac{x+y}{2} \cos \frac{x-y}{2}$$

$$\cos x + \cos y = 2 \cos \frac{x+y}{2} \cos \frac{x-y}{2}$$

$$\cos x - \cos y = -2 \sin \frac{x+y}{2} \sin \frac{x-y}{2}$$

$$\sin^2 \frac{x}{2} = \frac{1 - \cos x}{2}$$

$$\cos^2 \frac{x}{2} = \frac{1 + \cos x}{2}$$

$$\cos\left(\frac{\pi}{2} - x\right) = \sin x$$

$$\sin\left(\frac{\pi}{2} - x\right) = \cos x$$

Integral:

$$X_T = \frac{\int_K x dm}{m(K)} = \frac{\int_K x \rho ds}{\int_K \rho ds}$$

$$ds = |\dot{\vec{r}}(t)| dt = \sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2} dt$$

$$\int_K u ds = \int_a^b u(\vec{r}(t)) |\dot{\vec{r}}(t)| dt$$

$$l(K) = \int_K ds = \int_a^b |\dot{\vec{r}}(t)| dt$$

$$\int_K \vec{R} d\vec{r} = \int_K \vec{R} \cdot \vec{T} ds = \int_a^b \vec{R}(\vec{r}(t)) \cdot \dot{\vec{r}}(t) dt$$

$$\iint_{\Sigma} u ds = \iint_D u(\vec{r}(s,t)) \sqrt{EG - F^2} ds dt =$$

$$\iint_D u(\vec{r}(s,t)) |\vec{r}_s \times \vec{r}_t| ds dt$$

$$\iint_{\Sigma} \vec{R} d\vec{s} = \iint_D \vec{R} \cdot \vec{N} ds = \iint_D \vec{R}(\vec{r}) \cdot (\vec{r}_s \times \vec{r}_t) ds dt$$

$$\int_{\kappa} X dx + Y dy + Z dz = \int_{\kappa} (X, Y, Z) d\vec{r}$$

$$\iint_{\Sigma} X dz dy + Y dx dz + Z dx dy = \iint_{\Sigma} (X, Y, Z) d\vec{S}$$

$$\iint_D |\vec{r}_u \times \vec{r}_v| du dv = \iint_D \sqrt{EG - F^2} du dv$$

$$E = |\vec{r}_u|^2 \quad F = \vec{r}_u \cdot \vec{r}_v \quad G = |\vec{r}_v|^2$$

Površina grafa $f \in C^1(D)$

$$\iint_D \sqrt{1 + f_x^2 + f_y^2}$$

Površina torusa $0 < a < R$: $P = 2\pi a 2\pi R$

Gradient $\vec{\nabla} u = (u_x, u_y, u_z) = \text{grad } u$

divergencia $\vec{\nabla} \cdot \vec{R} = x_x + y_y + z_z = \text{div } \vec{R}$

rotor $\vec{\nabla} \times \vec{R} = (z_y - y_z, x_z - z_x, y_x - x_y) = \text{rot } \vec{R}$

$$S \xrightarrow{\text{grad}} V \xrightarrow{\text{rot}} V \xrightarrow{\text{div}} S$$

če naredimo dva zaporedna
koraka pride 0

$$\text{div}(\text{rot}(\vec{R})) = 0 \quad \text{rot}(\text{grad}(u)) = 0$$

$$\text{div} \circ \text{grad} = \Delta = \partial_x^2 + \partial_y^2 + \partial_z^2$$

Polje u je **harmonično** če $\Delta u = 0$

Polje $\vec{R}$ je **potencialno** če $\exists u$. $\vec{R} = \text{grad } u$

Polje $\vec{R}$ ima **vektorski potencial** če je
 $\vec{R} = \text{rot } \vec{f}$ za nek $\vec{f}$

Polje $\vec{R}$ je **irrotacionalno** če $\text{rot } \vec{R} = 0$

Polje $\vec{R}$ je **solenoidalno** če $\text{div } \vec{R} = 0$

$$\vec{f} = \text{grad } u \Rightarrow \int_K \vec{f} d\vec{r} = u(b) - u(a)$$

Gauss D omejena odprta

- rob iz končnega števila odsekov gladkih ploskev

$$\vec{R} \in C^1(\bar{D})$$



$$\oint_{\partial D} \vec{R} d\vec{s} = \iiint_D \operatorname{div} \vec{R} dV$$

Green D omejena odprta

- končno število odsekov gladkih krivulj za rob

$$x, y \in C^1(\bar{D})$$

$$\int_{\partial D} X dx + Y dy = \iint_D (Y_x - X_y) dx dy$$

Stokes Σ omejena odsekov gladke,

- rob iz končnega števila odsekov gladkih krivulj

$$\vec{R} \in C^1(\bar{\Sigma})$$

$$\int_{\partial \Sigma} \vec{R} d\vec{r} = \iint_{\Sigma} \operatorname{rot} \vec{R} d\vec{s}$$

Uporebno $\swarrow$ je skalarno

$$\text{grad } \frac{1}{|\vec{r} - \vec{a}|} = - \frac{\vec{r} - \vec{a}}{|\vec{r} - \vec{a}|^3}$$

$$\text{rot}(\vec{r} \times \vec{a}) = -2\vec{a}$$

$$\text{grad } \frac{\vec{a} \cdot \vec{r}}{\vec{b} \cdot \vec{r}} = \frac{\vec{r} \times (\vec{a} \times \vec{b})}{(\vec{b} \cdot \vec{r})^2}$$

$$\text{grad}(\text{div } \vec{f}) = \text{rot}(\text{rot } \vec{f}) + \Delta \vec{f}$$

$$\text{div}(\vec{f} \times \vec{g}) = \vec{g}(\text{rot } \vec{f}) - \vec{f}(\text{rot } \vec{g})$$

$$\begin{aligned} \text{rot}(\vec{f} \times \vec{g}) &= (\text{div } \vec{g})\vec{f} - (\text{div } \vec{f})\vec{g} - \\ &\quad (\vec{f} \text{ grad})\vec{g} + (\vec{g} \text{ grad})\vec{f} \\ &\quad (\vec{f} \text{ grad})\vec{g} = (\vec{f}(\text{grad } g_1), \vec{f}(\text{grad } g_2), \dots) \end{aligned}$$

$$\text{div}(u \vec{F}) = u \text{div } \vec{F} + \vec{\nabla} u \cdot \vec{F}$$

$$\text{div}(\vec{A} \times \vec{B}) = \vec{B} \text{ rot } \vec{A} - \vec{A} \text{ rot } \vec{B}$$

$$\iint_{\partial D} u \frac{du}{d\vec{n}} dS = \iiint_D (\text{grad } u \cdot \text{grad } u + u \Delta u) dV$$

$$\iiint_D \left(v \frac{du}{d\vec{n}} - u \frac{dv}{d\vec{n}} \right) dS = \iiint_D (u \Delta v - v \Delta u) dV$$

$D \subseteq \mathbb{R}^3$ je zvezdasto $\Rightarrow$

- $\vec{R}$ je potencialno $\Leftrightarrow \operatorname{rot} \vec{R} = \vec{0}$
(velja tudi za $\mathbb{R}^2$) $\uparrow$
- $\vec{R}$ ima vektorski potencial ($\exists \vec{G} \in C^2(\bar{D})$.
 $\vec{R} = \operatorname{rot} \vec{G}$) $\Leftrightarrow \operatorname{div} \vec{R} = 0$

• $\vec{G}$ je vektorski potencial $\Rightarrow \vec{G}_0 + \operatorname{gradu}$
je tudi vektorski potencial

• $\operatorname{div} \vec{G} = \operatorname{div}(\operatorname{gradu}) \Rightarrow \vec{G} = \operatorname{rot} \vec{F} + \operatorname{gradu}$

