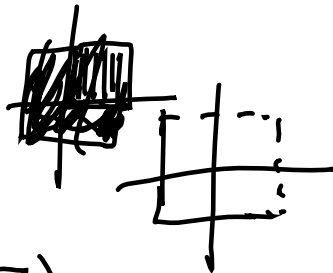


1.1.

$$A = [-1, 1] \times [-1, 1]$$

$$g: \mathbb{R}^2 \rightarrow \mathbb{R}^2 \quad \text{loc}$$



a) $g^*([-\infty, 0] \times [-\infty, 0])$

niada ni ti
niada ni ti
niada ni ti

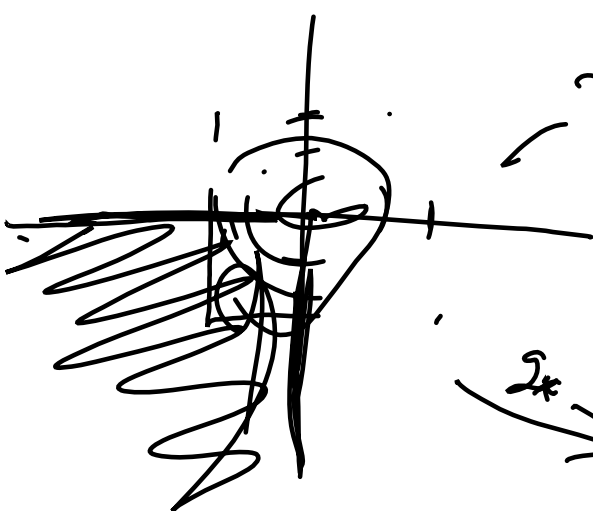
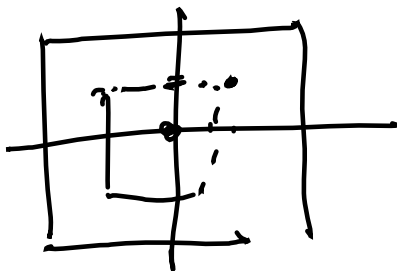
b) $g^*(\mathbb{R}^2 - (-\infty, 0] \times (-\infty, 0])$

niada ni ti
niada ni ti



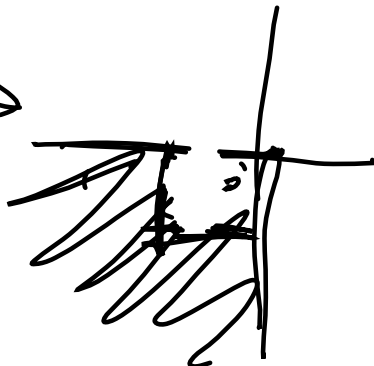
c) $g^*([-2, 2] \times [-2, 2])$

niada ni ti

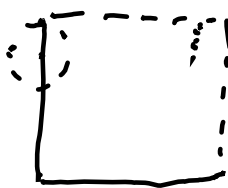
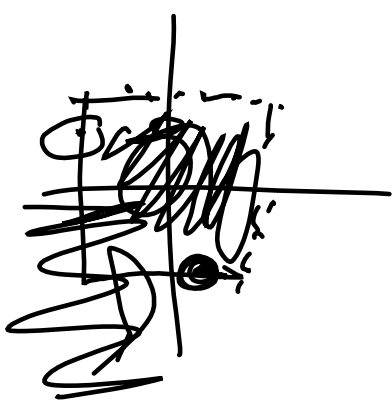


niada ni ti

g^*

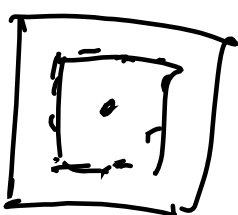


g^*

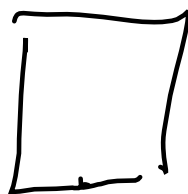


b)

$$g^*([-2, 2] \times [-2, 2])$$



$\rightarrow$



niada ni ti

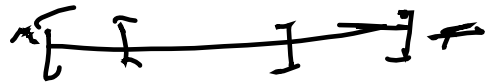
niada ni ti

c) niada ni ti

14)

$$a) [-2, 2] / [-1, 1]$$

$$\cong [-1, 1]$$



$$b) [-1, 1] / \{ -1, 0, 1 \}$$



$\cong$ ~~many~~

$$c) \mathbb{R}^n /$$

8

$$\mathbb{R}^n /_{K(0,1)} \xrightarrow{\approx} \mathbb{R}^n$$

$$\begin{array}{ccc} \mathbb{R}^n & \xrightarrow{f} & \mathbb{R}^n \\ \downarrow & & \\ \mathbb{R}^n /_{K(0,1)} & & \end{array}$$

$$f(z) := \begin{cases} 0; & \text{ce je } |z| \leq 1 \\ z - \frac{z}{\|z\|}; & z \in \mathbb{R}^n - \overline{K(0,1)} \end{cases}$$

$$d) \quad \mathbb{R}^n / \sim \quad x \sim y \Leftrightarrow \|x\| = \|y\|$$

$$\mathbb{R}^n / \sim \rightarrow [0, \infty)$$

$$f: \mathbb{R}^n \rightarrow [0, \infty)$$

$$f(\vec{x}) = \|\vec{x}\| \text{ je sur}$$

$$a \in [0, \infty) \quad (a, 0, 0, \dots) \mapsto a$$

$$[x] = [y] \Leftrightarrow \|x\| = \|y\| \Leftrightarrow f(x) = f(y)$$

je wznie

iścemo preslikavo s , da

$$voj \sim f \circ s = \text{id}_{[0, \infty)}$$

$$s: a \mapsto (a, 0, \dots)$$

Dokazimo da je $r \circ s = \text{id}_Y$

$\Rightarrow r$ kvocientna, surjekcija

$$S \subseteq Y \text{ takoda } r^*(S) \text{ odg } \forall X$$

$$S = s^*(r^*(S)) =$$

$$= (s^* \circ r^*)(S) = (r \circ s)^*(S) =$$

$$\text{id}_Y^*(S) = S$$

$\Rightarrow S$ je odg rta

f)

$$S^n \times [-1, 1] / \{S^n \times \{-1\}, S^n \times \{1\}\} \cong S^{n+1}$$

$$f: X \rightarrow S^{n+1}$$

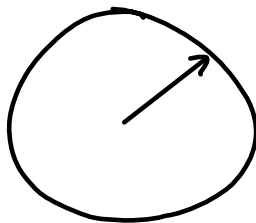
$$(x, t) \mapsto (xb, t) = (\sqrt{1-t^2}x, t)$$

↑
analoges vollen

$$\|xb\|^2 + \|t\|^2 = 1$$

$$\|b\|^2 + \|t\|^2 = 1$$

$$b = \sqrt{1 - \|t\|^2} = \sqrt{1 - t^2}$$



$$G/H = \{gH; g \in G\} =$$

$$= \{\{g \cdot h; h \in H\}; g \in G\}$$

G top. grupe

$$a \in G$$

$$L_a: G \longrightarrow G$$
$$x \longmapsto a \cdot x \quad \text{leva transkacija}$$

$$a, b \in G$$

$$h: G \longrightarrow G$$

$$h(a) = b \quad h = ?$$

$$L_{ba^{-1}}: x \longmapsto ba^{-1}x$$

možnost:

$$\begin{aligned} &ba^{-1}x \\ &x a^{-1}b \\ &b x a^{-1} \\ &a^{-1}x b \end{aligned}$$

topološke grupe gledajo

povsod isto, ker lahko vzeto

točke preslikamo v drugo s homeomorfizmom

2.1.

a)

$A \subseteq G$ desetica $\Leftrightarrow b a^{-1} A$ desetica $b \in G$

$$\exists U^{*} \subseteq A, a \in U$$

$$b \in b a^{-1} A$$

$$a \in U \Rightarrow b a^{-1} a \in b a^{-1} U \subseteq b a^{-1} A$$

" b

Ker je $L_{b a^{-1}}$ homeomorfizem je
 $b a^{-1} U \subseteq A$ odprta v G

$\Leftarrow$ potem velja tudi obratno

b) $H \subseteq G$ H delica $1 \Rightarrow H$ odprta v G

$$a \in aH \subseteq H$$

$\Rightarrow H$ je delica
vsake svoje točke

G/H je odprta.

$$aH \cap H = \emptyset \Rightarrow aH = H$$

$$a \in G/H \Rightarrow aH \cap H \neq \emptyset \Rightarrow$$

vsak element ima delico ki ne
seka $H \Rightarrow H$ je zaprta

c) C komponente $1 \in C$

$\Rightarrow C$ zaprta edinka v G

$$\underline{C \leq G}$$

$L_a: x \mapsto ax$ je homomorfizem za $\forall a \in C$

$$\underline{\forall a \in C. L_a \leq C}$$

L_a ohranja povezanost

$$\text{Pravi tako } L_{a^{-1}} = a \Rightarrow$$

$$L_a * C \cap C \Rightarrow L_a C \leq C$$

investiranje: investiranje je tudi homeo $i: x \mapsto x^{-1}$
 $\Rightarrow$ po istih argumentih ~~možna~~

Ali je edinka?

$$\underline{\forall a \in G. aC = Ca \Leftrightarrow aCa^{-1} \leq C}$$

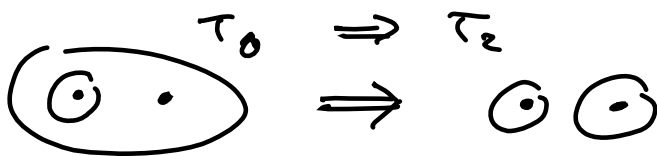
$x \mapsto axa^{-1}$ je homomorfizem

$\Rightarrow$ je povezana in $a \text{ id } a^{-1} = \text{id} \in C$

$$\Rightarrow aCa^{-1} \leq C$$

$\rightarrow$ ker je kompozitna tvel transkicij
(levo in desno)

d) τ_0 je



memo $L_{ba^{-1}}$

$\exists U \subseteq G \quad a \in U, b \notin U \quad \text{BŠZS}$

$$U^{-1} = \{a^{-1} ; a \in U\}$$

$$aU^{-1}b \ni b \quad a \mapsto b$$

Predmet da $a \in aU^{-1}b$

$$\exists c \in U, a = ac^{-1}b$$

$$\Rightarrow b = c \Rightarrow b \in U \quad \times$$

$$\tau_1 \Rightarrow \tau_2$$

$$\exists U, V \subseteq G, a \in U, b \in V, a \notin V, b \notin U$$

$\Delta \subseteq G \times G$ je zgrajen v $G \times G$

$$\Delta_0 = f^*(\Delta_1)$$

$$f: (k, x) \mapsto xy^{-1}$$

za vsaka praviho xy^{-1} je zgrajen

$$\textcircled{2} \quad T_{cc} \in T_1 \text{ in } n: T_2$$

$$\Rightarrow (\mathbb{R}, +) \text{ is a group on } T_{cc}$$

$$(\ker 1_d)$$

2.3) $\mathbb{Z} \times \mathbb{Z} \hookrightarrow \mathbb{R}^2$

a) $(m, n)(x, y) := (m+x, n+y)$

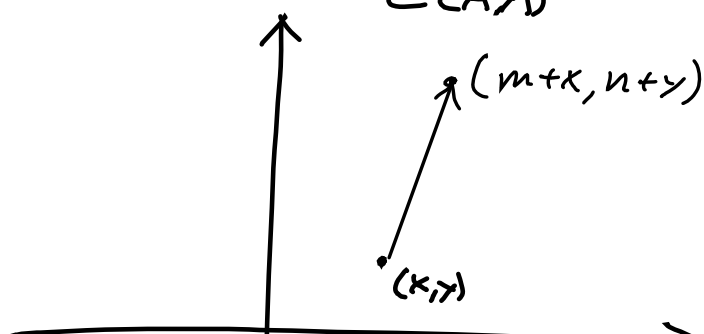
$\Phi: g \mapsto (a \mapsto g \cdot a)$

bijekcija (izomorfizam)

$\Phi: G \rightarrow \text{Bij}(A) \leftarrow \text{grupa}$

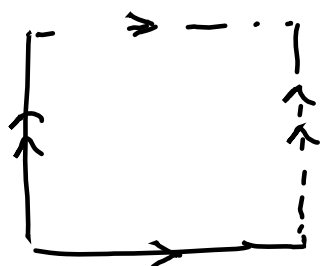
za kompoziciju

Ubišemo: $\Phi: G \rightarrow (\text{Konec}(A), \circ)$
 $\mathcal{L}(AA)$



memo $[0, 1) \cong \mathbb{R}^2 / \Phi$

Na nivou mrežice
 za nivou topologije? NE



Dobimo torus

 $\cong S^1 \times S^1$

$f: \mathbb{R}^2 \rightarrow S^1 \times S^1$

$(x, y) \mapsto (e^{2\pi i x}, e^{2\pi i y})$

Tak nismo resili već i

(2.3) b)

$$S_0 \times S_0 \hookrightarrow \mathbb{R}^2$$

$$(s, t) \cdot (x, y) = (sx, ty)$$

$$[0, \infty)^2$$

$$f: \mathbb{R}^2 \rightarrow [0, \infty)^2$$

$$f(x, y) = (|x|, |y|)$$

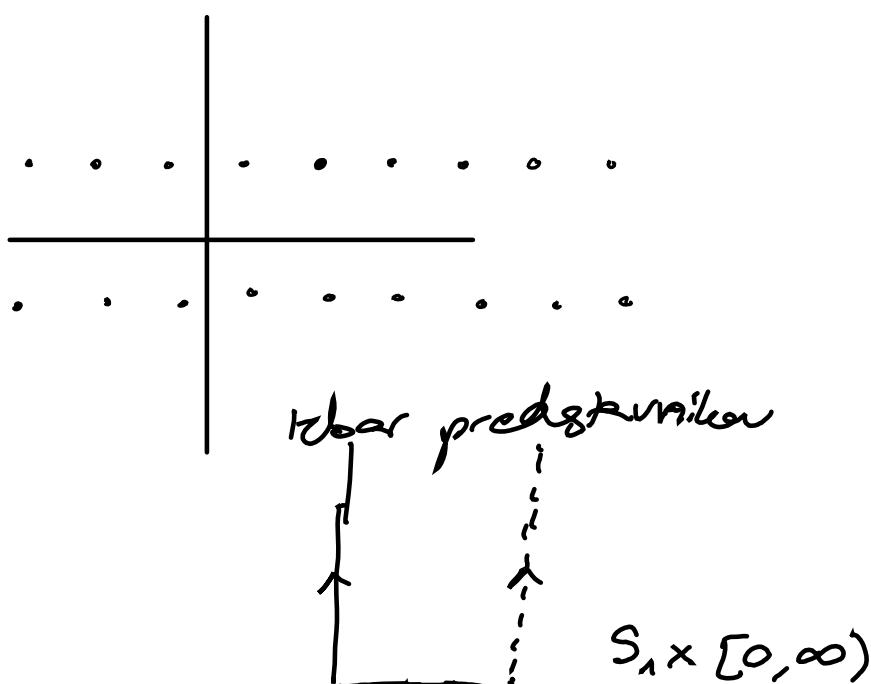
$$s: [0, \infty)^2 \rightarrow \mathbb{R}^2$$

$$(x, y) \mapsto (x, y)$$

f, s zusammen $\Rightarrow f$ Inverse

$$c) \mathbb{Z} \times S^1 \hookrightarrow \mathbb{R}^2$$

$$(m, t) \cdot (x, y) = (m+x, ty)$$



$$f: \mathbb{R}^2 \rightarrow S^1 \times [0, \infty)$$

$$(x, y) \mapsto (e^{i2\pi x}, |y|)$$

zema $\checkmark$
surjektiven $\checkmark$

Po standardnem postopku tudi identifikacije med delavarije

$$\left(\left(n - \frac{1}{n} \right), 0 \right)_n$$

Slike zaporedja zap v $\mathbb{R}^2$
f-slike pa ni zap v $S^1 \times [0, \infty)$

(1,0) je v zaprtju nra v f-sliki

Produkt dveh adyrtih preslikov je adyrt

$$h: \mathbb{R} \longrightarrow [0, \infty)$$

$$x \mapsto |x|$$

Dovolj prevesti na bazo

$$0 \notin (a, b): h(a, b) = |\min \{ |a|, |b| \}, \max \{ |a|, |b| \} \}$$

$$\text{če } 0 \in (a, b): h(a, b) = [0, \max \{ |a|, |b| \}]$$

$$g: \mathbb{R} \rightarrow S^1 \subseteq \mathbb{C}$$

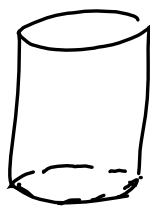
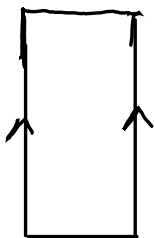
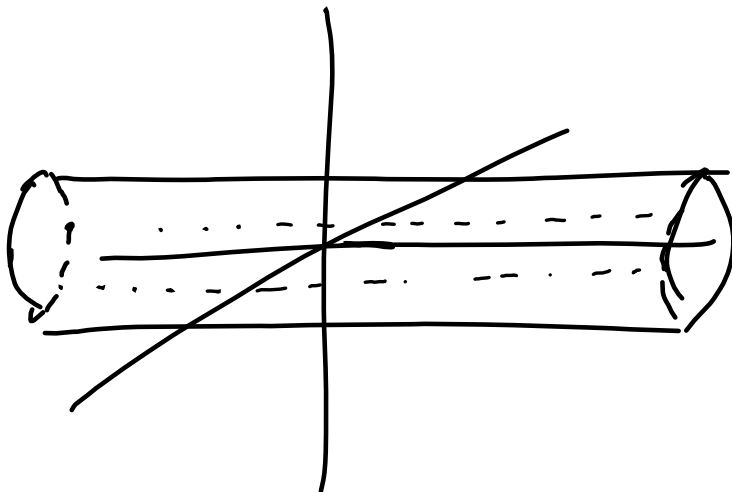
$$x \mapsto e^{i2\pi x}$$

Baza 2 a_1, a_2 i intervali b_1, b_2 $\in \mathbb{C}$ $\Rightarrow$
slike
adyrt lok

d)

$$\mathbb{Z} \times S^1 \hookrightarrow \mathbb{R} \times S^1 \subset \mathbb{R}^3$$

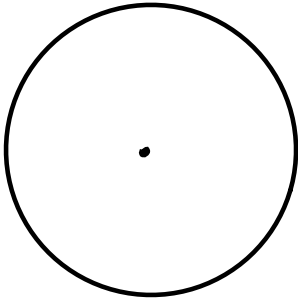
$$(m, t), (x, y, z) := (m+x, y, tz)$$



$$\mathbb{R} \times S^1 \longrightarrow S^1 \times [1, 1]$$

$$(t, \frac{y}{z}) \longmapsto (e^{2\pi i t}, y)$$

e
10)



$$\vec{x} \sim \vec{y} \Leftrightarrow \|\vec{x}\| = \|\vec{y}\|$$

$$f: \vec{x} \longrightarrow \|\vec{x}\|$$

???

Tepelgij - ke - steen

$$f: \mathbb{R}^2 \longrightarrow \mathbb{R}^1$$

A retrakcija $\Rightarrow$ koeficijenti v
• zgoraj or neke

2(4)

$$2x = g + x$$

$\mathbb{R}/\mathbb{Q}$ n: matrici v. l. o. z. z. v. n. o. n. n.

$$U^{\infty} \subseteq \mathbb{R}/\mathbb{Q} \quad \leftarrow \text{po dell'azione gruppo}$$

Ubit v. s.

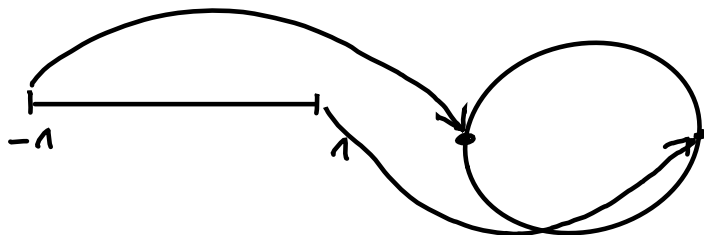
3.1

8.4

a)

$$X = [-1, 1] \quad A: \{-1, 1\}$$

$$Y = S^1 \quad f(x) := (x, 0)$$



$$Z = [-1, 1] \times \{0\} \cup S^1$$

$$g: X + Y \longrightarrow Z$$

$$i_1(x) \longmapsto (x, 0)$$

$$i_2(y) \longmapsto y$$

ekvivalenci; razredi: $(i_1(1,0), i_2(1,0))$ in $i_1(-1,0)$ in $i_2(-1,0)$ neenoteno

$$g(i_1(1,0)) = 1,0$$

$$g(i_2(1,0)) = 1,0$$

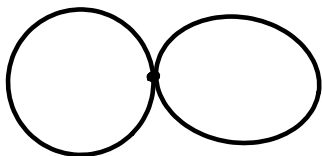
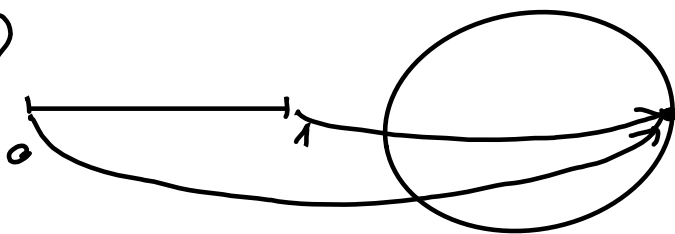
padeta na 2. razr.

zvenost pa, ker sta preslikavi zven;
in ker ~~se~~ se ajineta na preslik

Potrebo se pokaže, še da loči dvi. razrede

Sikamo iz kompaktna v Hausdorffa

b)



$$Z = Y \cup S((2,0), 1)$$

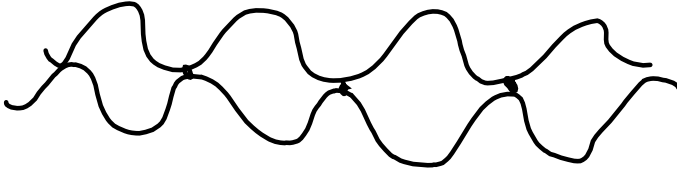
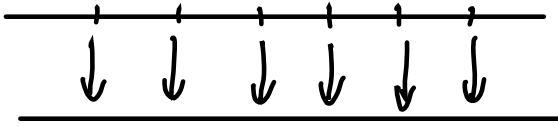
$$g: X+Y \longrightarrow Z$$

$$\text{in}_1(x) \longmapsto (\sin \pi x, \cos \pi x)$$

$$\text{in}_2(x,y) \longmapsto (-x+2, y)$$

Preveriti moramo

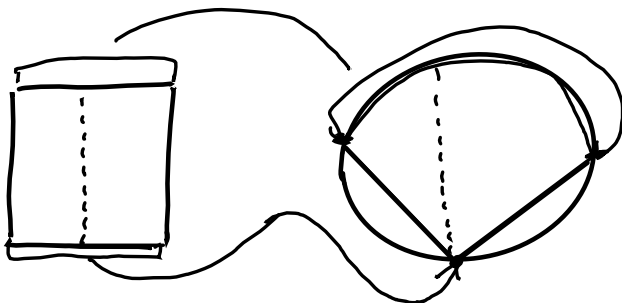
- loči dve raze
- konšt na dve raze
- vez, surj, kvocientne v ožjem smislu



$$|\sin x| \cup -|\sin x|$$

$x \mapsto$

d)



$$Z = S^1 \cup \{(x, y) \in \mathbb{R}^2; |x-1| \leq 1 \leq \sqrt{1-x^2}\}$$

$$\partial: x+y \longrightarrow Z$$

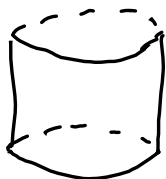
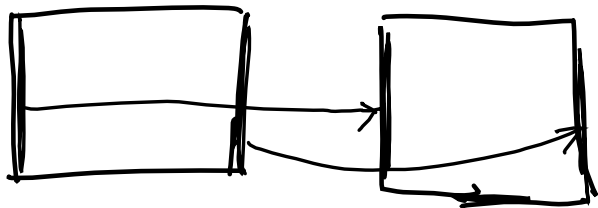
$$\text{in}_2(z) \longmapsto z$$

$$\text{in}_1(x, y) \longmapsto (0, -1) + \frac{y+1}{2} (x, \sqrt{1-x^2}+1)$$

kompakte v hausdorff

e)

$X:$



$$Z = S_1 \times [-1, 1]$$

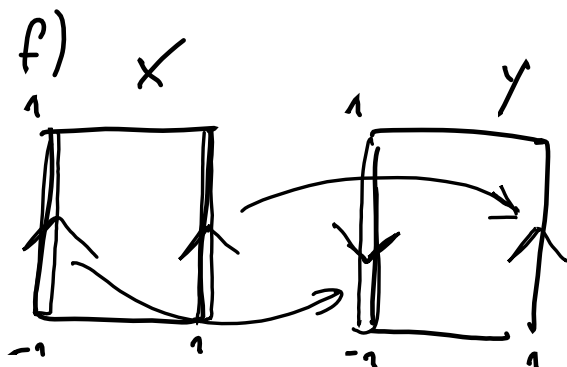
$$g: X + Y \longrightarrow Z$$

$$in_1(x, y) \longmapsto x(\sqrt{1-x^2}, y)$$

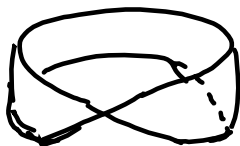
$$in_2(x, y) \longmapsto (x, \sqrt{1-x^2}, y)$$

kompakt & hausdorff ✓

also ist identifizierung



Möbiusstreifen



Parametrisierung des Möbiusstreifen
 $in_1(u, v) \mapsto (x, y, z)$

$$x(u, v) = \left(\cos \frac{v\pi}{2} (2 + u \cos \frac{v\pi}{4}) \right)$$

$$y(u, v) = \left(\sin \frac{v\pi}{2} (2 + u \cos \frac{v\pi}{4}) \right)$$

$$z(u, v) = u \sin \frac{v\pi}{2}$$

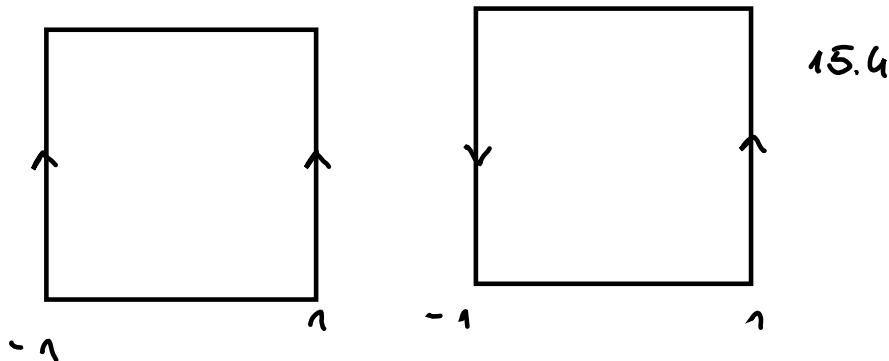


$in_2(u, v) \mapsto (x, y, z)$

$$x = \cos \frac{v\pi}{2} (2 + u \cos \frac{v\pi}{4})$$

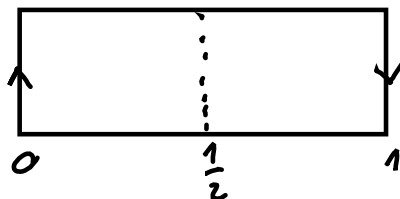
$$y = -\sin \frac{v\pi}{2} (2 + u \cos \frac{v\pi}{4})$$

$$z = -u \sin \frac{v\pi}{2}$$



$X + Y \longrightarrow M \dots$ Möbius dreieck

$$M = [0, 1] \times [0, 1] / \sim$$



$$\begin{array}{ccc}
 X + Y & \xrightarrow{\delta'} & [0, 1]^2 \\
 \downarrow 2 & \searrow g & \downarrow \sim \\
 X \cup_f Y & \dashrightarrow & M
 \end{array}$$

$$in_1(u, v) \mapsto \left(\frac{u+1}{4}, \frac{v+1}{2} \right)$$

$$in_2(u, v) \mapsto \left(\frac{3-u}{4}, \frac{v+1}{2} \right)$$

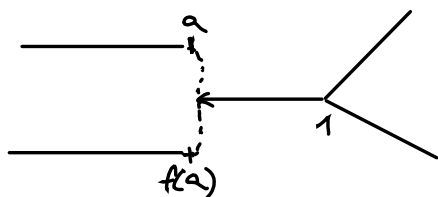
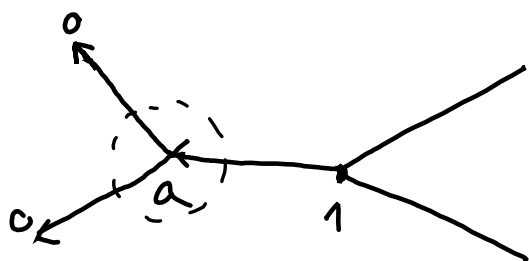
$$x \sim y \iff g(x) = g(y)$$

$$x \sim y \iff g'(x) \sim g'(y)$$

$$(a, 1] \longrightarrow (0, \infty)$$

$$f(x) = x$$

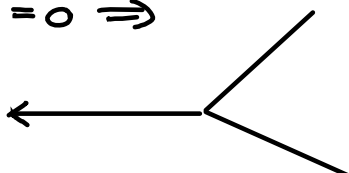
zlepsek $(0, \infty) \cup (0, \infty)$ je zlepsek za
kake hausdorffove



Poglejmo točko a

Vseke dolice od a in od $f(a)$ seka te
del $(0, 1]$ za $\forall a \in (0, 1)$

$$\text{če } a = 0 \Rightarrow$$



Vložno v evklidski prostor

$$(0, \infty) \cup (0, \infty) \longrightarrow \mathbb{R}^2$$

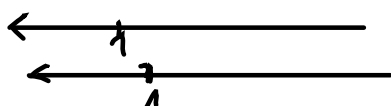
$$x \in (0, 1] \longmapsto (0, x)$$

$$x \in (1, \infty) \longmapsto (x, x-1)$$

$$x \in (1, \infty) \longmapsto (x, -x+1)$$

je hausdorffov?

ima kompaktnost



kompleti: zaprti intervali v $(0, \infty)$

v 1: zaprt interval $[\frac{1}{2}, \frac{3}{2}]$ na obeh
premnicah

kompleti groz v kompaktnosti

(3.3)

$$X/A \approx X \cup_f 1$$

$$\begin{array}{ccc} X & \xleftarrow{\alpha'} & X + 1 \\ f \downarrow & \searrow \alpha & \downarrow \alpha \\ X/A & \xrightarrow{\approx} & X \cup_f 1 \end{array} \quad 1 \in 1$$

h:

$$\begin{array}{ll} [x] \mapsto x & : x \notin A \\ [a] \mapsto 1 & : a \in A \end{array}$$

is homeomorphism

$$\begin{aligned} g' : X + 1 &\rightarrow X \\ \text{in}_1 x &\mapsto x \\ \text{in}_2 a &\mapsto 1 \quad a \in A \end{aligned}$$

$$g = f \circ g'$$

$$u \sim v \Leftrightarrow g'(u) \sim_A g'(v)$$

$$u \sim v \Leftrightarrow g(u) = g(v) \Leftrightarrow$$

$$f(g'(u)) = f(g'(v)) \Leftrightarrow$$

$$[g'(u)] \sim [g'(v)]$$

Then recall: $v \in X \cup_f 1$

$$\text{in}_1 x = \{x\} \quad x \notin A$$

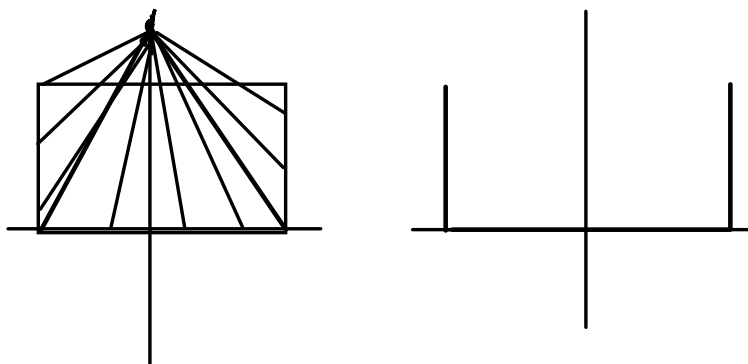
$$\text{in}_1 a = A \cup \{a\} \quad a \in A$$

$$\text{in}_2 a = A \cup \{a\}$$

$$[\text{in}_1 x] = \{ \text{in}_1 x \}$$

$$(\text{in}_1)_*(A) \cup \{ \text{in}_2 a \}$$

Retrakti, homotopije, ekvivalenca i negativni loop



Projekcija iz $(0,2)$ na rob

$$y > 2x+2 : (-1, 2 - \frac{y-2}{a})$$

$$2x+2 > y < -2x+2 : (\frac{2x}{2-y}, 0)$$

$$y > -2x+2 : (1, \frac{y-2}{x} + 2)$$

$$(a,b) \leadsto p_1: y = \frac{b-2}{a}x + 2$$

$$\text{pri } x = -1 \\ y = -\frac{b-2}{a} + 2$$

$$p_2: y = \frac{b-2}{a}x + 2 \quad a \neq 0$$

$$y=0 \rightarrow x = \frac{-2a}{b-2} = \frac{2a}{2-b}$$

$$x = 1:$$

$$y = \frac{b-2}{a} + 2$$

Ali je to id na y ?

$$x = -1: (-1, 2 - \frac{y-2}{-1}) = (-1, 2 + y - 2) = (-1, y)$$

$$y = 0: (\frac{2x}{2}, 0) = (x, 0)$$

$$x = 1: (1, \frac{y-2}{1} + 2) = (1, y) \checkmark$$

Ali homotopije id na X

$$H(x, y, t) = t(x, y) + (1-t)r(x, y)$$

zvana, he so vsi kosi zvezna

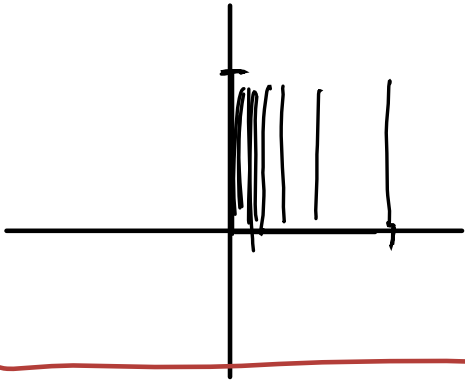
f homotop g

$$\text{ko } \exists H: X \times [0, 1] \longrightarrow Y$$

$$(x, 0) \longmapsto f(x)$$

$$(x, 1) \longmapsto g(x)$$

Wann



Refrakt T_2 prostora je zaprt

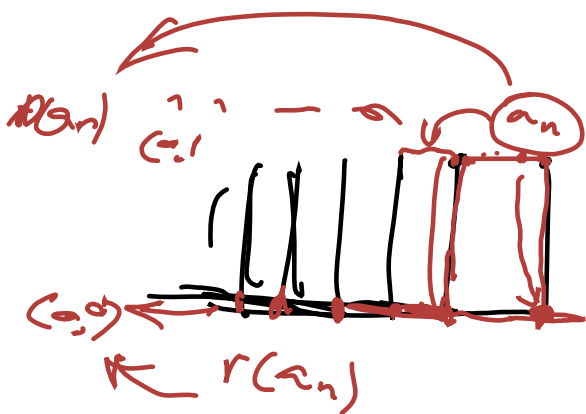
Recimo da $\exists r$ rebrakt $: X \rightarrow Y$
 $r/x = i/a$

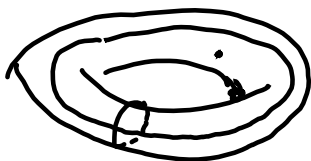
deljica $[\frac{1}{n+1}, \frac{1}{n}] \times \{1\}$

morajo pakati točke $(\frac{1}{n}, 0)$

ker gre vsaka p+ iz $(\frac{1}{n+1}, 1)$ do $(\frac{1}{n}, 1)$
 torej $\text{če } z \text{ koda}$

Naj bo zaporedje a_n ; $r(a_n) = \frac{1}{n}$
 $a_n \in [\frac{1}{n}, 1] \times \{0\}$
 $\lim a_n = (0, 1)$
 $\lim r(a_n) = (0, 0)$
 $r \nmid a_n$: wenz





$$G \sim X$$

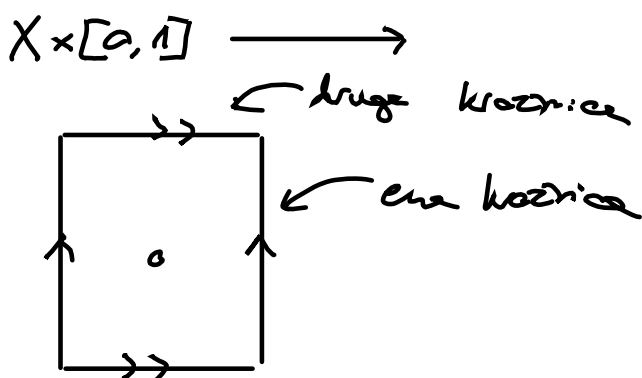
$$(x, 0) \longrightarrow x$$

$$(x, g) \longmapsto x \circ g$$

Deformacijske relacije "ima homotopiju do id" je jedna preslika $H: X \times [0, 1] \rightarrow X$.

$$H(x, 0) = x \quad , \quad H(a, 1) = a \quad \wedge \quad H(x, 1) \in A$$

$$\forall x \in X \quad \forall a \in A \quad \forall x \in X$$



$$([-1, 1]^2 - \{0, 0\}) \times [0, 1] \xrightarrow{h} [-1, 1]^2 - \{0, 0\}$$

$$\downarrow \begin{matrix} \times \text{id}_{[0, 1]} \\ \text{vezna preslika} \end{matrix} \quad \searrow \text{goh} \quad \downarrow g$$

$$X \times [0, 1] \longrightarrow X$$

$$h: (x, t) \longmapsto \frac{\vec{x}}{\|\vec{x}\|_\infty} t + \vec{x} (1-t)$$

goh mora biti konstantna na ekv. razdel

$$x \sim y \Rightarrow goh(x) = goh(y)$$

$$\Leftrightarrow [h(x)] = [h(y)]$$

h ekvivalentne relacije definirane na množici

$$(x_1, y_1, t_1) \sim (x_2, y_2, t_2) \in [-1, 1]^2 - \{0, 0\} \times [0, 1]$$

$$\Leftrightarrow t_1 = t_2 \quad \wedge \quad [x_1, y_1] = [x_2, y_2]$$

$$\Rightarrow \frac{(x_1, y_1)}{\|(x_1, y_1)\|_\infty} t_1 + (x_1, y_1) (1-t_1)$$

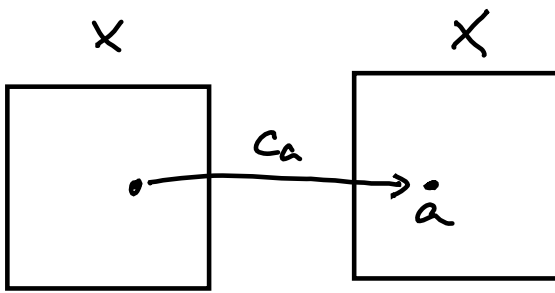
$$\sim \frac{(x_2, y_2)}{\|(x_2, y_2)\|_\infty} t_2 + (x_2, y_2) (1-t_2)$$

Izrek

q kvocientna in X kompakten T_2 prostor
 $\Rightarrow q \times id_X$ je kvocientna

Boj splošno:

q kvocientna, X lokalna kompaktna
 $\Rightarrow q \times id$ kvocientna



$\Rightarrow$ naj bo X povezan s p.d.m.:

Naj bo γ te pot med a in b po glasu

$$\gamma(0) = a \quad \gamma(1) = b$$

$$H: X \times [0, 1] \rightarrow X$$

$$(x, t) \mapsto \gamma(t)$$

$$H = \gamma \circ \text{pr}_2 \text{ torej je vera}$$

$\Leftarrow$

Recimo, da $\forall a, b \in X$ velja $c_a \approx c_b$

Naj bosta a, b poljubna

$$\exists H: X \times [0, 1] \rightarrow X$$

$$H_0 = c_a$$

$$H_1 = c_b$$

$$\gamma(t) = H(a, t)$$

45

a) $f: S^n \rightarrow S^n$ $n: \text{sur}$

homotopie kvasditi

$a \neq b \notin f_*(S^n)$ b je nespriti tej

$$H: S^n \times [0, 1] \rightarrow S^n$$

$$(x, t) \mapsto \frac{(1-t)f(x) + t \cdot a}{\|(1-t)f(x) + t \cdot a\|}$$

Ali je sučajno $(1-t)f(x) + t \cdot x = 0$

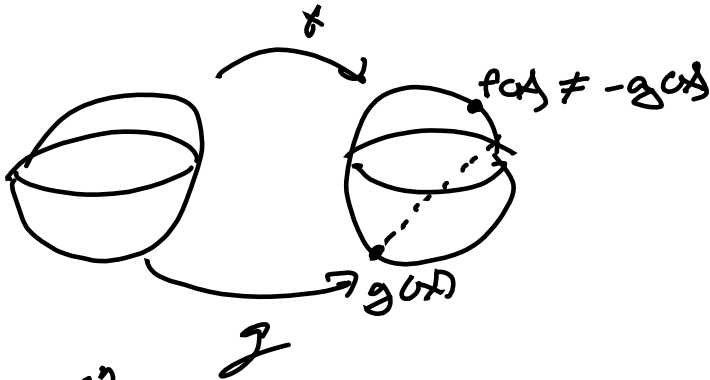
$\Rightarrow$ če greš le delica skozi 0

ampak a ni zlogi vrednosti, zato

nikoli ne gre delica skozi 0,0

$$b) f, g : S^n \rightarrow S^n$$

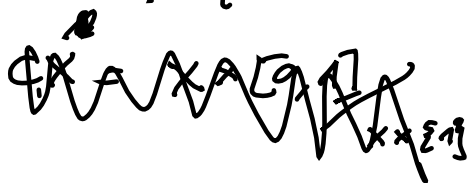
$$f(x) \neq -g(x) \quad \forall x \in S^n \Rightarrow f \approx g$$



$$H : S^n \times [0, 1] \rightarrow S^n$$

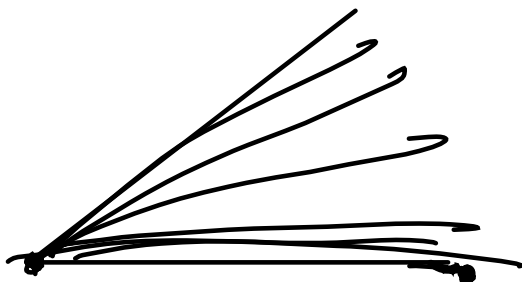
$$(x, t) \mapsto \frac{(1-t)g(x) - t f(x)}{\|(1-t)g(x) - t f(x)\|}$$

$$4.6 \quad X = \{ [0, 1] \times \{0\} \} \cup \{ x, \frac{x}{n}, x \in [0, 1] \text{ in } \mu \}$$



$$H: X \times [0, 1] \longrightarrow CX$$

$$(x, y), t \longmapsto (1, t)(x, y)$$



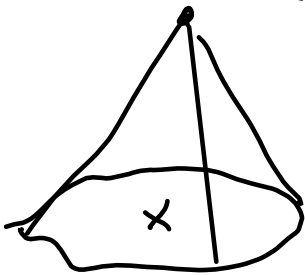
Rečimo da je H G poljubna nepravilna deformacija retrakcije x na neko točko

$$\forall n \in \mathbb{N}. \exists t_n \in [0, 1] \quad G\left(1, \frac{1}{n}, t_n\right) = (0, 0)$$

ker je $[0, 1]$ kompaktna lahko izberemo konvergenčni podskup t_n

?

$$CX = X \times [0, 1] / \sim$$



$$CX \times [0, 1] \longrightarrow CX$$

$$X \times [0,1] \times [0,1] \xrightarrow{H} X \times [0,1]$$

$$\begin{array}{ccc} \downarrow \pi \circ id & & \downarrow \pi \\ CX \times [0, 1] & \dashrightarrow & CX \end{array}$$

$H: (x, u, t) \mapsto \begin{matrix} \text{no more update} \\ \text{constant} \\ \cancel{(x, 1)t + (x, u)(1-t)} \\ (x, u(1-t) + t) \end{matrix}$

Preveriti maksimuma samo

$$(x_1, 1, t) \sim_H (x_2, 1, t)$$

etini nedrviahi du; ualench razea

$$(x_1, 1) \quad \parallel \quad (x_2, 1)$$

ste ekvivalentni ker sta na v. 1 in 1

$$\bar{H}([x, u, \sigma]) = [x, u]$$

$$g(H(x, u, \phi)) = g(x, u)$$

$$\mathfrak{g}(H)(x, u, 1) = \dots$$

X je kontraktilen



$\exists$ nema izbira pti od poljubne točke X
do izbrane točke

$$\exists a \in X. \exists \gamma: X \rightarrow \mathcal{C}(I, X), \gamma(x)(0) = x, \gamma(x)(1) = a$$



$X \neq \emptyset \wedge \exists$ nema izbira pti med točkami: $\forall x$

$$\exists f \gamma: X \times X \rightarrow \mathcal{C}(I, X), f(a, b)(0) = a, f(a, b)(1) = b$$

u.8 (u.7 ne učimo)

i) $\Rightarrow$ ii) Vemo iz u.7 (u.6)

ii) $\Rightarrow$ iii) Naj bo X rešetki kontraktilnega prostora Y

$$\exists r: Y \rightarrow X. \exists i: X \rightarrow Y. r \circ i = id$$

(ima zvezni dani inverz (ekvivalentna definicija retrakcije))

$$\exists y_0 \in Y. \exists H: Y \times I \rightarrow Y$$

$$(y, 0) \mapsto y$$

$$(y, 1) \mapsto y_0$$

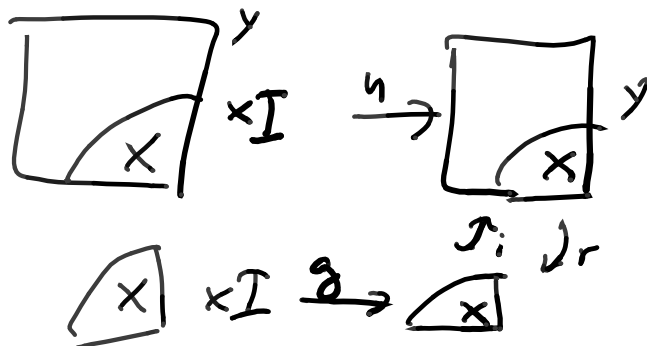
izamo

$$g: X \times I \rightarrow X$$

$$x_0 = r(y_0)$$

$$g(x, 0) = g$$

$$g(x, 1) = x_0$$



$$g(x, t) = r(h(i(x), t))$$

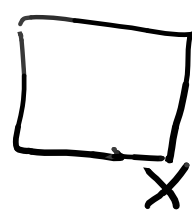
$$g = r \circ h \circ (i \times id_I)$$

iii) $\Rightarrow$ i)

$$\square \times I \xrightarrow{H} \square \cdot a \times X$$

$$H(x, 0) = x$$

$$H(x, 1) = a$$



$$r \circ i = id$$

$$r([x, 0]) = x$$

$$CX = X \times I / \{x, 1\}$$

$$[x, t] \mapsto p_1(H(x, t))$$

u.9

$$I \times I /_{\text{veno}} = M$$

$$\text{veno} \Leftrightarrow \sim$$

CM

$$I \times I /_{\text{veno}} \times I /_{\{[0,1]\}} = \text{CM}$$

Recimo da je kontraktilen

$$\exists H: I \times I /_{\text{veno}} \times I \rightarrow I \times I /_{\text{veno}}$$

$$H(x, 0) = x$$

$$H(x, 1) = 1$$

veam:mo podprostor $A = \{x \in I \times I /_{\text{veno}} \mid x \text{ je na robu}\}$

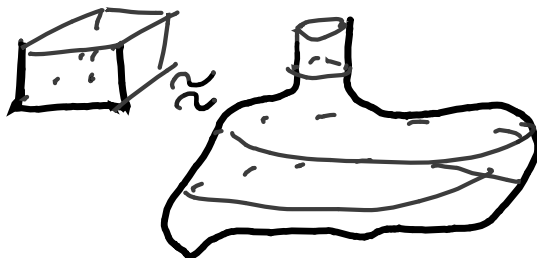
ki je krožnica topološka

torčje je tudi A kontraktilen

u. 10

Megezen konveksen prostor je
kontraktilen

Splazno: zvezdast prostor je kontraktilen



$\mathcal{N}$ - Razred normalnih topoloških prostov

$y \in AE(\mathcal{N})$ je absolutni distenzor za vrh normalnih prostov

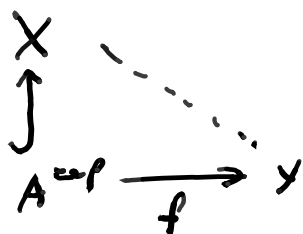
kadar za $\forall X \in \mathcal{N}$ in vsako podmnožico

$A \subseteq X$ velja, da lahko vsake preslikave

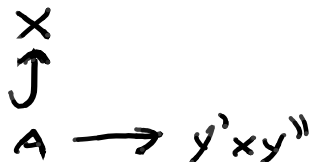
$A \rightarrow Y$ razširimo do neke preslikave

$X \rightarrow Y$

Torej: $IEAE(\mathcal{N})$



• Produkt AE je AE



razširimo v vsaki komponenti

• Retrakt AE je AE

• $X \in AE(\mathcal{N}) \Rightarrow X \neq \emptyset$ in X povezan s pdm:

• $AE(\mathcal{N}) \cap \mathcal{N} \subseteq AR(\mathcal{N})$

↖ absolutni retracts

Ne tabl: 4. ~~1824~~ 11

4. 12

$$A, B \subseteq \mathbb{R}^n$$



$$\mathbb{R}^n \in \mathcal{A}$$

Dukejme $A \cup B$ je rektat $\mathbb{R}^n$

Falsche

4.19

$$v \in \mathbb{R}^n \setminus \{0\}$$

$$Av = \lambda v$$

$$\text{bšzs } \|v\| = 1$$

$$\|Av\| = \|\lambda v\| = |\lambda| \dots = \lambda$$

$$Av = \lambda v \stackrel{?}{=} \|Av\| v$$

$$v = \frac{Av}{\|Av\|}$$

iščemo najbližje točko preslikave
 $S^{n-1} \rightarrow S^{n-1}$

$v \mapsto \frac{Av}{\|Av\|}$ ne nujno dobro definirana
 preslikava, ker
 $\ker A$ ni nujno $\{0\}$

$$X = \{ (x_1, \dots, x_n) \in S^{n-1} \mid x_1, \dots, x_n \geq 0 \} \approx B^{n-1}$$

če $v \in X$ so vsi $x_i \geq 0$ in niso vsi $= 0$

$\Rightarrow Av$ ima pozitivne komponente,
 vsakej $\neq 0$

$B^{n-1} \rightarrow B^{n-1}$ na najbližje točko

Mnogoterost

a) $\{\frac{1}{n}, n \in \mathbb{N}\}$ skupnost + diskretnost
= je mnogoterost

b) $\{\frac{1}{n}, n \in \mathbb{N}\} \cup \{0\}$

$\forall 0$ ni mnogoterost

če bi jo imela bi namreč bili $\mathbb{R}^0 = \{0\}$
ali $\mathbb{R}_+^0$ kar ne drži.

Torej tudi o množici rečemo
veliko stvari, toda od a) to pomeni:

ni o biti zbiranja, kar pomeni:

Skupina vsota n-mnt je n-mnt

0 dim

1 dim

20.5

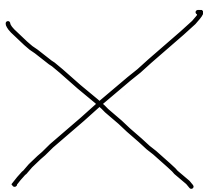
$$c) \quad x \in \{ -1, 1 \} \times \mathbb{R} \quad x \in \mathbb{R} \times \mathbb{R}$$

Produkt množic je množica

$$\Rightarrow 1 \text{ množica}$$

Tudi: Ste na vsaki n-množici je množica

d)



n: množica v $(0,0)$

$(1,1)$ ima 1-množico na delu

Torej v $(0,0)$ mora biti 1 množica

Tudi na n : $x \in \mathbb{R}$ ali $\mathbb{R}_+$

nej bo U poljubna delica tebe

Ni dovolj preverjati na bozi

$U = \{ (0,0) \}$ ima vsej 4 komponente

če bi U bil homeomorfen $\mathbb{R}$ ali $\mathbb{R}_+$

bi razdel na 1 ali 2 komponente

$\Rightarrow$ ~~ni množica~~ U ni množica

Naredimo vložitev tdn množici L in
 raba in se predkava n: od tega p den
 ni množica

$$f: \mathbb{R} \rightarrow X$$

$$f(x) = (x, x)$$

$f(\mathbb{R})$ mora biti od tega

$(0,0)$ je raba

toda se dlo

ker vsebuje $(\varepsilon, -\varepsilon)$ za
 dovolj majhen $\varepsilon > 0$
 vse delice

$$c) (\mathbb{R}^2 \times \{0\}) \cup (S^1 \times \mathbb{R})$$

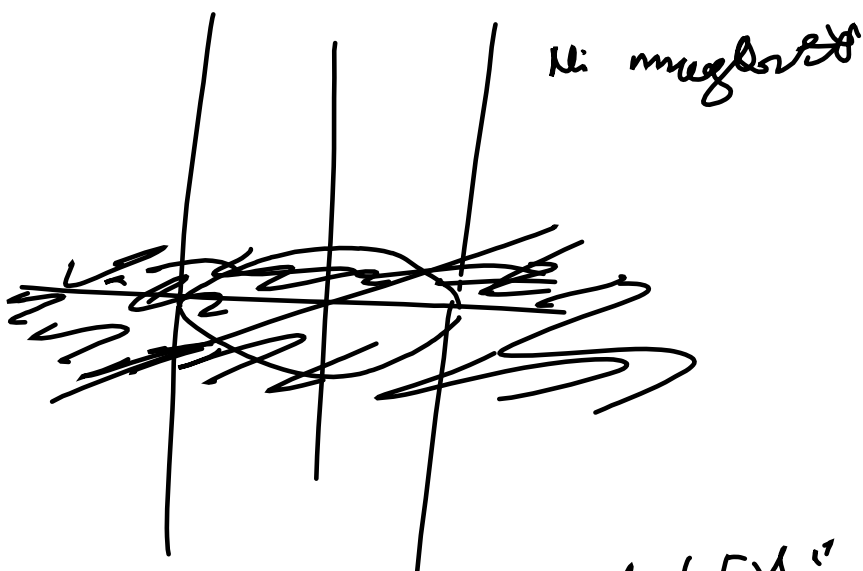
(za 1 nalogo)

Izrek o invarianci odprtih množic za mnogoterosti

če sta M, N istodimenzionalni mnogoterosti

M brez roba in $f: M \rightarrow N$ zvezna mekeja,

potem je f odprta vložitev v int N



ker bi bil vložitev

$$f: \mathbb{R}^2 \rightarrow X$$

$$x \mapsto (x, 0)$$

$f_*(\mathbb{R}^2)$ ni odprta ker

$(1, 0, 0)$ je robna za SL^2

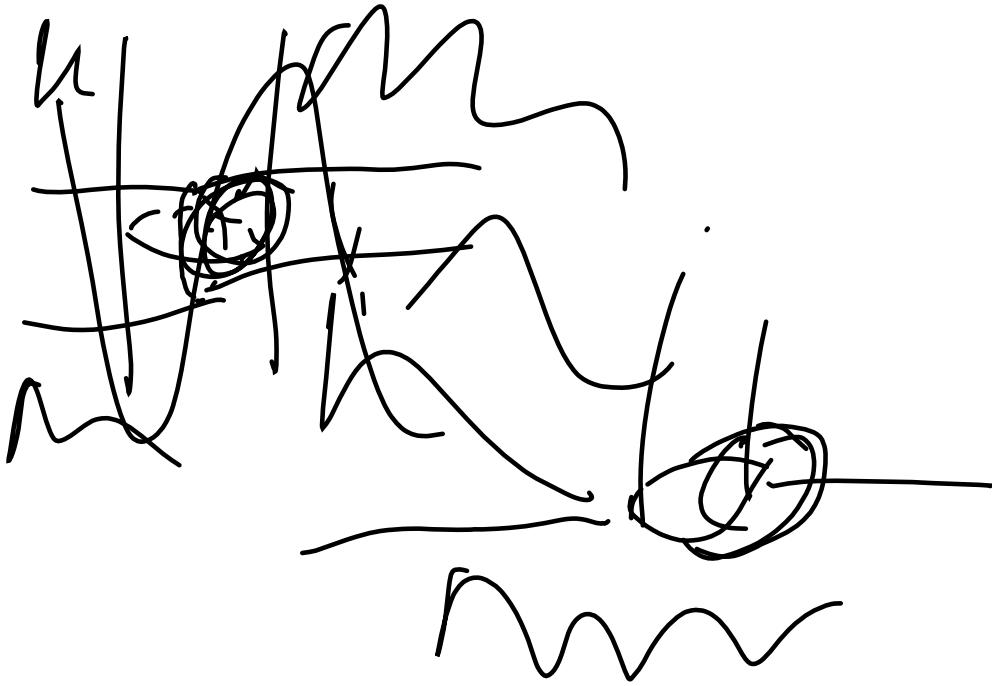
$$\forall U \ni (0, 0, 0)$$

$$(1, 0, \varepsilon) \in U$$

(malo bi dalo
zračnost
(kot v (0,0))

0,0; med delice
 $K \setminus \{0, \frac{1}{2}\} \times \mathbb{R}^2$
 $\mathbb{R}^2 \ni \frac{1}{2} \in \mathbb{R}^2$

f)



voorkeur: $\leftarrow$ 2 meetpunten

$$f: S^1 \times \mathbb{R}^3 \rightarrow X$$

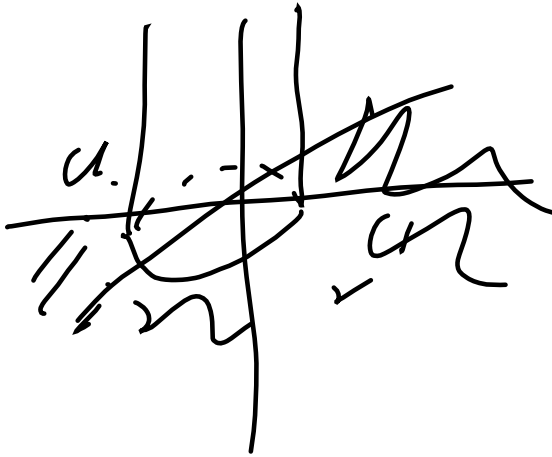
$$x \mapsto x$$

Waarvoor $(1, 0, 0)$

$$\partial(M \times N) = \partial M \times N \cup M \times \partial N$$

$(1, 0, 0)$ is voor de 2e devaluering

g)



$\mathbb{P}^1$ & n ~~ungetrennt~~

$$\approx S^1 \times \mathbb{R}$$

$$A: X \rightarrow S^1 \times \mathbb{R}$$

$$x = re^{i\varphi}, r \geq 0$$

$$f: X \mapsto X \quad \text{~~for } x \geq 0 \quad x \mapsto x+1~~}$$

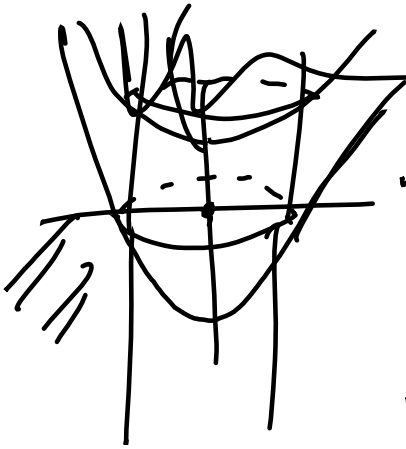
$$(re^{i\varphi}, 0) \mapsto (e^{i\varphi}, r+1)$$

$$r=1 \Rightarrow e^{i\varphi}, r-1+1 = \text{seu } \varphi \text{ neue Elementare}$$

invers

$$e^{i\varphi}, z \mapsto \begin{cases} e^{i\varphi}, z & z \geq 0 \\ (z+1)e^{i\varphi}, 0 & z < 0 \end{cases}$$

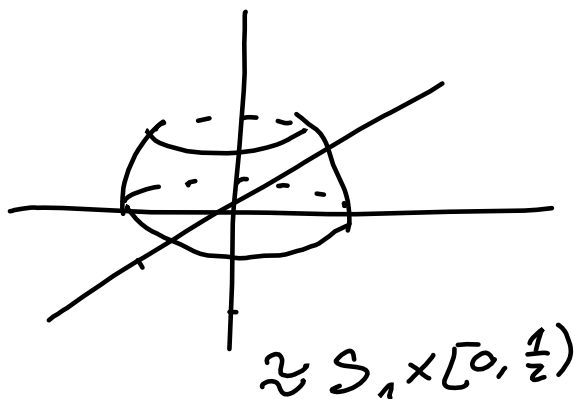
h)



ni ~~ti~~ ~~de~~ ~~domene~~
~~ma~~ ~~restato~~

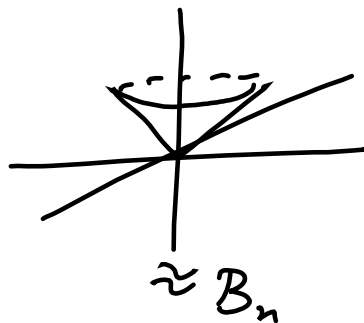
Visto per

5.2



ima
lastnosti
negativne točke
crkve je za 90°

Rob ima
2 komponenti



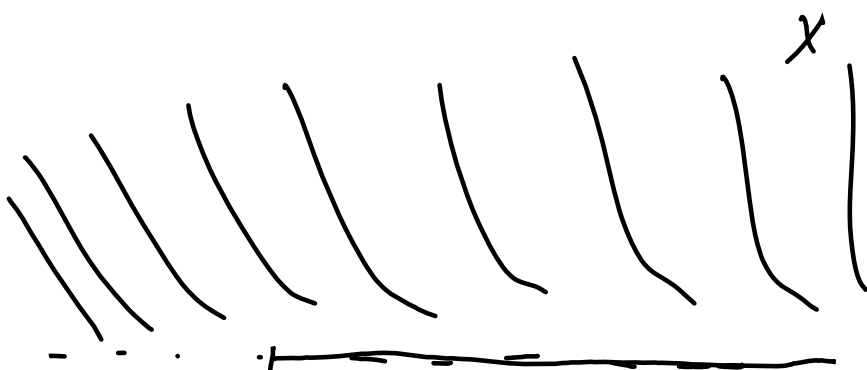
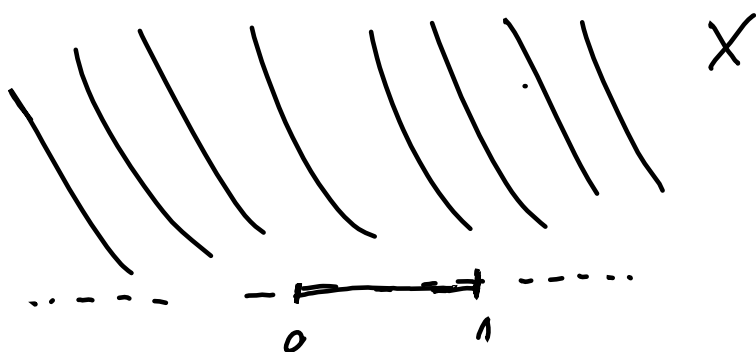
↓
pr optičke rje

→
prijelaz

ima
lastnosti
negativne točke

Rob ima
ena komponenta

5.3



X ima 2 namrečlostni točki
 Y pa ima 1

$v(0, \infty)$ ni množica ker ni
 lokalno kompaktna v tej točki

$Z = \mathbb{R} \times [0, \infty)$ ker v točki si konvergirajo

$Z \rightarrow X$ v notranjost
 $Z \rightarrow Y$ v notranjost

~~Reči~~ domene $[0, \infty)$ ose v

$$Z \hookrightarrow X \xrightarrow{h} Y \hookrightarrow \mathbb{R}^2$$

zanimanje se stika v notranjost

$h_*(Z)$ je odsek pot $\mathbb{R}^2$

in $h_*(Z) \subseteq Y \subseteq \mathbb{R}^2$ v Y

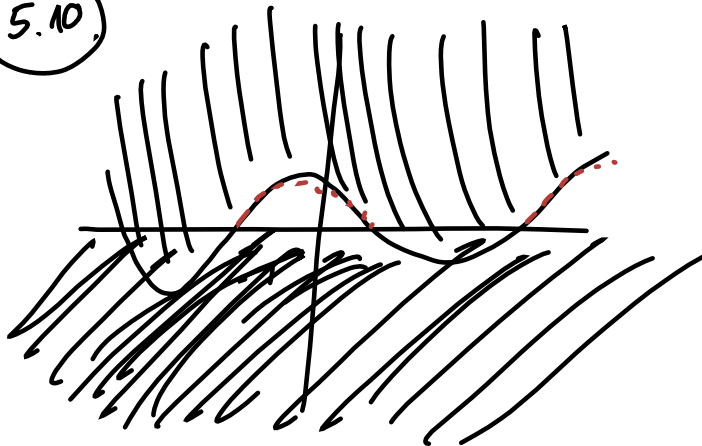
$$\Rightarrow h_*(Z) \subseteq Z$$

je eden glavnih

$$(h^{-1})_*(Z) \subseteq Z$$

$\Rightarrow h|_{[0, 1] \times \{0\}} \rightarrow [0, \infty) \times \{0\}$ je detektor
 nenulni X

5.10.



a) $p_{n-1}(x) = x^2 - 1$



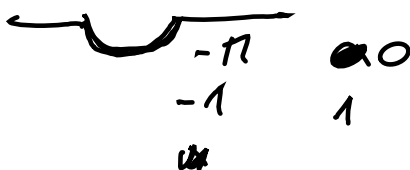
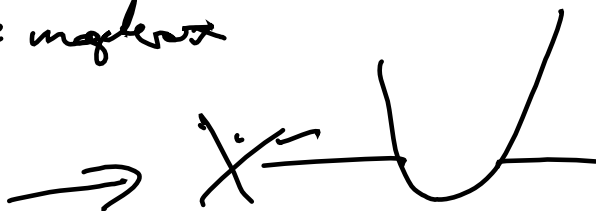
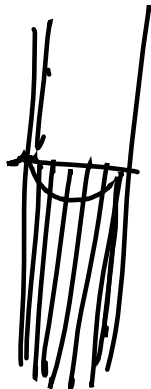
X_{p-1} ni lok konjugata

$\vee (-1, 0)$

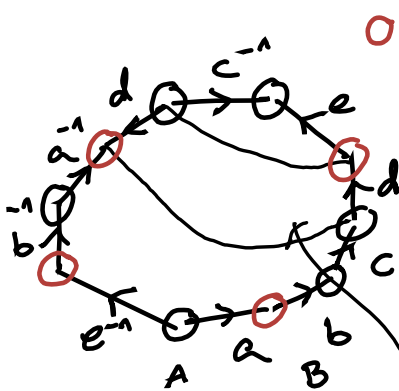
kerc se vedlo daly ∞

$\exists$ zaporedje $(-1, \varepsilon_n)$

X_{p-1} je mogleva



$x, y \mapsto (y + \sqrt{1-x^2}) e^{i \frac{(x+1)\pi}{2}}$



0 in 0 sta sene

ker se pogov:

veleke oke
nebarke drakot
je + m negde ost

Ni arentebike, ker j mabrusar drak

0-celice: 1

1-celice: 5

2-celice: 1

$$\chi(X) = -3$$

ker je neorientebike bo to nP

$$\chi(nP) = 2 - n$$

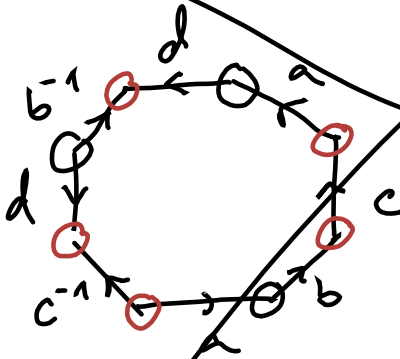
$$2 - n = -3$$

$$n = 5$$

$$\chi(nT) = 2 - 2n$$

$$X \approx 5P$$

je ploštev
neorientirane
ni sklenjena

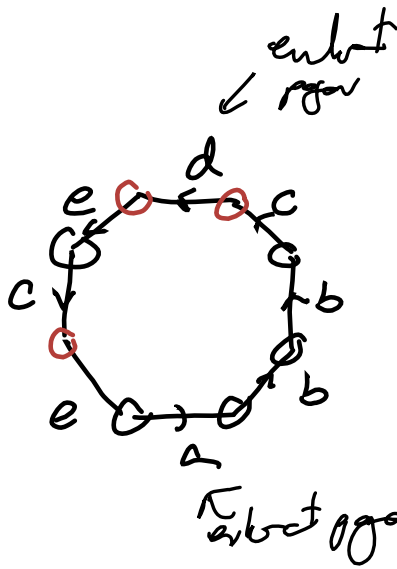


0-celice: 2

1-celice: 5

2-celice: 1

nepravilna
naloge ::



enl
komp. ploskev

n : sklenjena

n : orientabilna

$$\chi(D) = 2 - 5 + 1$$

5 robovih komponent: 2

Kombinacija

$D' = D$ kjer na vsako robno komponento prilepimo disk:

$\Rightarrow$ dodamo 2 2-celice:

0-celice: 2

1-celice: 5

2-celice: 3

$$\chi(D') = 0$$

nP :

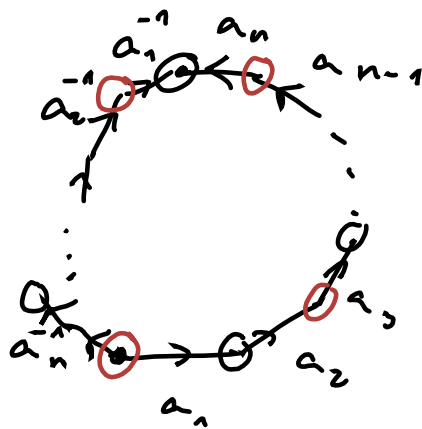
$$2 - n = 0$$

$$n = 2$$

$$D' = 2P$$



$\Rightarrow D$ je $2P$ ~~komponent~~
z dvema luknjama



kompletne
orientebrana
sklenjena

n sodo: 0-celice: 1

n liho: 0-celice: 2

1-celice: n

2-celice: 1

n sodo: $\chi(G) = 1 - n + 1 = 2 - n$

n liho: $\chi(G) = 2 - n + 1 = 3 - n$

$$2 - 2m = 2 - n$$

$$2 - 2m = 3 - n$$

$$2m = n$$

$$m = \frac{n}{2}$$

$$m = \frac{n-1}{2}$$

$$G \cong \frac{n}{2} T$$

$$G \cong \frac{n-1}{2} T$$

5.14

a) sistem enot:

$$X \# Y \approx 2T$$

$$X \# T \approx Y \# K$$

$$\delta(X \# Y) \approx \delta X + \delta Y$$

X, Y sklenjeni
orientabilni

X neorientabilna
 $\Rightarrow$ ni ~~ne~~ $\Rightarrow$

b) $X \# Y \# Z \approx 2K \# T$ ← sklenjena plošča

$$X \# Y \approx Z \# 2T$$

$$X \# Z \approx Y \# K$$

X ali Z neorientabilna

X, Y, Z sklenjeni

Z neorientabilna $\Rightarrow X$ ali Y sta neorientabilna

$$\chi(X \# Y) = \chi(X) + \chi(Y) - 2$$

$$X + Y - 2 + Z - 2 \approx \chi(4P) + \chi(T) - 2$$

$$X + Y + Z - 4 \approx 2 - 4 + 2 - 2 - 2$$

$$X + Y + Z = 0$$

$2K + T$ je neorientabilna $\Rightarrow 2 - n = 0$

$$X \# Y \# Z \approx 2P$$

$$X + Y - 2 = Z + 2 - 2 - 2$$

$$X + Y - Z = 2 - 4 = -2$$

$$X + Z - 2 = Y + 2 - 2$$

$$X - Y + Z = 2$$

$$2Y = -2$$

$$Y = -1$$

$$2Z = 2$$

$$Z = 1$$

$$X = 0$$

Orientabilne
plošče imajo
sodo χ

Z neorientabilna $\Rightarrow$

$$X \approx 3P$$

$$Z \approx P$$

Y je lahko orientabilna
ali ne

$Y \approx T$ je orientabilna

$Y \approx 2P$ je neorientabilna

$\parallel$
 K

$$X \# M \approx Y \# P$$

π_0 je aditivno

$$X \# Y \approx M \# N$$

$$\chi(M \# N) = \chi M + \chi N \uparrow$$

$$\pi_0(\partial Y) = 1 + \pi_0(\partial X)$$

$\uparrow$ št komponent za povezanost s polni; = št robov in komponent

$$\pi_0(\partial X) + \pi_0(\partial Y) = 1$$

$$\pi_0(\partial X) \neq 1 + \pi_0(\partial X) = 1$$

$$\pi_0(\partial X) = 0 \Rightarrow \pi_0(\partial Y) = 1$$

X sklenjena

Möbiousov trak = projekтивna ravnina brez enega diska

$$\chi(M) = \chi(P) - 1 = 0$$

$$X + Y - Z = 0 + 2 - 3 - 2$$

$$X + 0 - Z = Y + 1 - 2$$

$$X - Y = 1$$

$$X + Y = -1$$

$$X = 0$$

$$Y = -1$$

$$X \approx T \vee X \approx 2P$$

$$X \approx T \Rightarrow Y \approx T$$

5.15

a)

orientabilna

$\chi(A)$

$$\chi(X \cup \text{Trak}) = \chi(X) - 1$$



$$B^n \cup_{\partial B^n} B^n \cong S^n$$

$$B^n + B^n \longrightarrow S^n$$

$$in_1: x_1, \dots, x_n \mapsto (x_1, \dots, x_n, \sqrt{1-x_1^2-x_2^2-\dots-x_n^2})$$

$$in_2: x_1, \dots, x_n \mapsto (x_1, \dots, x_n, -\sqrt{1-x_1^2-\dots-x_n^2})$$

uzemi:

prave identifikacije:

$$\|x_1, \dots, x_n\| = 1 \mapsto x_1, \dots, x_n, 0$$

$$\mapsto x_1, \dots, x_n, 0$$

je konstant na dva razreda

$$\text{Če } \vec{r} = \sqrt{1-|r|} = r, -\sqrt{1-|r|}$$

$$\text{poker } \vec{r} = \vec{r}$$

$$\sqrt{1-|r|} = -\sqrt{1-|r|}$$

surjektivnost

$$\Rightarrow 1-|r| = 0$$

$$|r| = 1$$

$$f: S^{n-1} \rightarrow S^{n-1}$$

$$F: B^n \rightarrow B^n \quad F|_{S^{n-1}} = f$$

$$F(\vec{r}) = \begin{cases} \|\vec{r}\| f(\frac{\vec{r}}{\|\vec{r}\|}) \\ 0 & ; \vec{r} = 0 \end{cases}$$

$$\lim_{t \rightarrow 0} \|t \cdot e\| f\left(\frac{t \cdot e}{\|t \cdot e\|}\right) = \lim_{t \rightarrow 0} t \cdot f(e) = 0$$

$$c) D_1 \cup B^n \approx B_2$$

$$X \approx S^n$$

$$X = D_1 \cup D_2 \rightarrow S^n$$

$$h_1: D_1 \rightarrow D_n \quad \text{hmm.}$$

$$h_2: D_2 \rightarrow B_n$$

$$x \mapsto \begin{cases} h_1(x) & ; \sqrt{1-h_2(x)^2} \\ h_2(x) & -\sqrt{1-h_2(x)^2} \end{cases} \quad \text{u: dobra definicija}$$

$$\partial D_1 \xrightarrow{h_1|_{\partial D_1}} S_{n-1}$$

$$\partial D_2$$

$$S_{n-1} \xrightarrow{\sim} \partial D_1 \xrightarrow{h_1|_{\partial D_1}} S_{n-1}$$

dobro je F da velja $B_n \rightarrow B_n$

$$F: B_n \rightarrow B_n$$

$$F|_{S^n}: \checkmark$$

$$F \circ (h_2^{-1}):$$

$$F \circ (h_2^{-1}): D_2 \rightarrow B_n$$

$$x \mapsto \begin{cases} h_1(x), \sqrt{1-h_2(x)^2} \\ F \circ (h_2^{-1}(x)), -\sqrt{1-h_2(x)^2} \end{cases}$$

