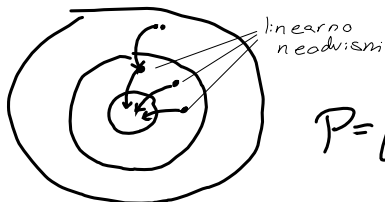


Jordanova formula $\ker(A-\lambda I) \subset \dots \subset \ker(A-\lambda I)^m$

$\dim \ker(A-\lambda I) \dots$ št celic za λ (geom. vec)

$m \dots$ velikost največje celice

$\dim \ker(A-\lambda I)^k - \dim \ker(A-\lambda I)^{k-1} \dots$ št celic velikosti
več kot $k \times k$



$$P = [(A-\lambda I)^{m-1} v_1, (A-\lambda I)^{m-2} v_1, \dots, v_1, \dots, v_2, \dots]$$

$$z = \begin{bmatrix} z_1 \\ \vdots \\ z_n \end{bmatrix} \quad f(z) = \begin{bmatrix} f(z_1) \\ \vdots \\ f(z_n) \end{bmatrix}$$

$$f(J) = \begin{bmatrix} f(\lambda) & f'(\lambda) & \dots & \frac{f^{(n-1)}(\lambda)}{(n-1)!} \\ & \ddots & \ddots & \vdots \\ & & f'(\lambda) & \\ & & & f(\lambda) \end{bmatrix}$$

Vektorska polja

Tokovnica vektorskega polja je krivulja γ , ki
reši $\dot{\gamma}(t) = F(\gamma(t))$ $F: D \rightarrow \mathbb{R}^n$
 $\gamma: I \rightarrow \mathbb{R}^n$

Tok vektorskega polja je $\Phi: \mathbb{R}^n \times \mathbb{R} \rightarrow \mathbb{R}^n$
 τ_+
 $\frac{\partial}{\partial t} \Phi_t(x) = F(\Phi_t(x)) \quad \Phi_0(x) = x$

Trditev: V vek. polje na $D \subseteq \mathbb{R}^n$. $V(p) = 0$ za nek $p \in D$

tokovnica γ ki zadošča $\gamma(0) = p \quad \gamma \equiv p$

Po eksistenčnem izreku je to edina tokovnica

Linearni sistem:

Homogen sistem: $\dot{x} = Ax$

Fundamentalna rešitev: $\Phi(t) = e^{At} = P e^{J^t} P^{-1}$

Splošna rešitev: $\Phi(t) x_0$

$$\dot{\Phi} = A \Phi \quad \Phi(0) = id$$

Nehomogen sistem: $\dot{x} = Ax + f(t) \quad (\Phi(-t) = \Phi^{-1}(t))$

Rešitev $x = x_n + x_p \quad \dot{x}_n = A x_n$

$$x_n = e^{At} x_0$$

$$x_p(t) = \int_0^t \Phi(t) \Phi^{-1}(s) f(s) ds$$

$$x_p = e^{At} C(t) \quad \leadsto e^{At} \dot{C}(t) = f(t)$$

$$C(t) = \int_0^t e^{-As} f(s) ds$$

$$x_p = e^{At} C(t) = e^{At} \int_0^t e^{-As} f(s) ds$$

Nasveti:

- imaginarne lastne vrednosti? uvedba novih koordinat

- $x_0 = \sum v_i$; v_i lastni vektorji: nastavek: $x(t) = \sum e^{\lambda_i t} v_i$

- splošni nastavek $x_j(t) = \sum e^{\lambda_i t} p_i(t)$

$p_i(t)$ polinom stopnje največ večkratnosti λ_i ;
(v karakt. polinomu)

(velja tudi za kompleksne)

(za rešitve linearnega sistema s konst. koeficienti)

Lineerne enačbe n-tega reda

$$a_0 y^{(n)} + \dots + a_n y = 0$$

nastavek: $y(x) = e^{\lambda x}$

Dobimo $p(\lambda) = a_0 \lambda^n + \dots + a_n = 0$

$\lambda_1, \dots, \lambda_n$ paroma različne $\Rightarrow$

$$y(x) = C_1 e^{\lambda_1 x} + \dots + C_n e^{\lambda_n x}$$

večkratna ničla (stopnje k) μ

$$b_0 e^{\mu x} + b_1 x e^{\mu x} + \dots + b_n x^{k-1} e^{\mu x}$$

$\lambda = a + bi \leadsto A e^{ax} \cos(bx) + C e^{ax} \sin(bx)$

Eulerjeva LDE (rede n)

$$a_0 x^n y^{(n)} + \dots + a_n y = 0$$

nastavek: $x = e^t$

$$z(t) = y(e^t)$$

$$\Rightarrow a_0 z^{(n)} + \tilde{a}_1 z^{(n-1)} + \dots + \tilde{a}_n z = 0$$

$$z(t) = p(t) e^{\lambda t} \leadsto y(x) = p(\ln x) x^\lambda$$

$\lambda = a + bi \leadsto x^\lambda = A x^a \cos(\ln b) + B x^a \sin(b \ln x)$

Nehomogene enačbe

$$a_0 y^{(n)} + \dots + a_n y = e^{\mu x} p \quad \text{rešitev: } y_h + y_p$$

y_p rešitev $\Rightarrow$ vse rešitve so oblike $y_h + y_p$

$$\text{nastavek } y_p = e^{\mu x} g(x) x^k$$

2... polinom stopnje kot p

k ... kratnost μ v $p(\lambda)$ (al: 0 če μ n: ničla)

Eulerjeva DE

$$a_n x^n y^{(n)} + \dots + a_0 y = x^\mu p(\ln x) \quad p(\lambda) = \sum_{k=0}^n a_k \frac{\lambda!}{(\lambda-k)!}$$

$$\text{Nastavek: } y_p = x^\mu g(\ln x) (\ln x)^k$$

g ... iste stopnje kot p

k ... kratnost μ v $p(\lambda)$

Variacija konstante za LDE

$$y_h = C_1 y_1(x) + \dots + C_n y_n(x)$$

$$y_p = C_1(x) y_1(x) + \dots + C_n(x) y_n(x) \quad \swarrow \text{odvodi od } C_j$$

$$\Phi \begin{bmatrix} c_1 \\ \vdots \\ c_n \end{bmatrix} = \begin{bmatrix} y_1 & \dots & y_n \\ y_1' & \dots & y_n' \\ \vdots & & \vdots \\ y_1^{(n-1)} & \dots & y_n^{(n-1)} \end{bmatrix} \begin{bmatrix} c_1 \\ \vdots \\ c_n \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ f/a_0 \end{bmatrix}$$

Determinanta Wronskega

$$W(t) = \det \Phi(t) = \begin{vmatrix} y_1 & y_2 & \dots & y_n \\ y_1' & \dots & \dots & y_n' \\ \vdots & & & \vdots \\ y_1^{(n-1)} & \dots & \dots & y_n^{(n-1)} \end{vmatrix}$$

Liouvilleov izrek: $W'(t) = \text{tr}(A(t))W(t)$

$$W(t) = W(t_0) e^{-\int_{t_0}^t \text{tr}(A(s)) ds}$$

u, v lin. neod. rešitvi: $y'' + p(x)y' + g(x)y = 0 \Rightarrow W = -u'v - uv'$

$W' + pW = 0 \Rightarrow W = e^{-\int p dt}$

$$\begin{vmatrix} u & v \\ u' & v' \end{vmatrix}$$

Če sta u, v lin. neodvisni; v eni; točki sta linearno neodvisni;

Vemo: $y'' + p y' + g y = 0$ p, g zvezni na $I \subset \mathbb{R}$ intervalu

1. $\Rightarrow$ vse ničle od rešitve u so enostavne

2. $\Rightarrow$ množica ničel u nima stekelščev I

3. $\Rightarrow x_1 < x_2$ $u(x_1) = u(x_2) = 0 \Rightarrow \exists x_3 \in (x_1, x_2) : v(x_3) = 0$

Vemo: $y'' + p y' + g y = 0$

1) $W' + pW = 0$

2) $W(x) = W(x_0) e^{-\int_{x_0}^x p(t) dt}$

p, g zvezni;
funkciji

3) $v = u \int \frac{W}{u^2} dx$

VARIACIJSKI RAČUN

$$A \subseteq C^2([a, b])$$

$$\mathcal{L}: A \rightarrow \mathbb{R} \quad L: [a, b] \times \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R} \quad C^2$$

$$\mathcal{L}(y) := \int_a^b L(x, y(x), y'(x)) dx$$

Smerni; odvod: $D\mathcal{L}_y(h) = \left. \frac{d}{dt} \right|_{t=0} \mathcal{L}(y+th)$

$$Q\mathcal{L}_y(h) = \left. \frac{d^2}{dt^2} \right|_{t=0} \mathcal{L}(y+th)$$

$$F: \mathbb{R}^n \rightarrow \mathbb{R} \in C^2 \Rightarrow DF = \nabla_x F(x)$$

$$QF = h^T H_F(x) h = \langle h, H_F(x) h \rangle$$

$$\mathcal{A} = \{y \in C^2([a, b]); y(a) = A, y(b) \in B\}$$

$$y+th \in \mathcal{A} \Leftrightarrow h(a) = 0 \wedge h(b) = 0$$

$$D\mathcal{L}_y(h) = \int_a^b (L_y(x, y, y') - \frac{d}{dx} L_{y'}(x, y, y')) h(x) dx$$

$$y_0 \text{ je ekstrem} \Leftrightarrow D\mathcal{L}_{y_0}(h) = 0 \Leftrightarrow L_y - \frac{d}{dx} L_{y'} \equiv 0$$

Euler-Lagrangeov
pogoj

$$L = L(y, y') \quad \mathcal{L}(y) \text{ ima kritično točko v } y \Leftrightarrow L - y' L_{y'} \equiv C \in \mathbb{R}$$

$$Q\mathcal{L}_{y_0}(h) = \int_a^b (h^2 (L_{yy} - \frac{d}{dx} L_{yy'}) + h'^2 L_{y'y'}) dx$$

Bertramjeva identiteta

$$y_0 \text{ kritična} \wedge Q\mathcal{L}_{y_0}(h) > 0 \Rightarrow y_0 \text{ je minimum}$$

$$y_0 \text{ kritična} \wedge Q\mathcal{L}_{y_0}(h) < 0 \Rightarrow y_0 \text{ je maksimum}$$

Popustne variacije so tiste za katere velja

$$y+th \in \mathcal{A}$$

$\mathcal{A}$ pa je podan z robnimi

$$\text{pogoji: } (y+th)(1) = 0$$

oblike

Keristo

$$C = \int \frac{1}{(1+x^2)^2} dx \leadsto C = A \arctan x + B \ln(1+x^2) + \frac{Px+E}{1+x^2}$$

$$\operatorname{ch}^2 t = \frac{\operatorname{ch} 2t + 1}{2}$$

$$\operatorname{ch}^2 t - \operatorname{sh}^2 t = 1$$

$$\operatorname{ch} 2t = \operatorname{ch}^2 t + \operatorname{sh}^2 t$$

$$\operatorname{sh} 2t = 2 \operatorname{sh} t \operatorname{ch} t$$

$$\operatorname{arsh} t = \ln |x + \sqrt{1+x^2}|$$

Nehomogeni sistemi s konst. koeficienti

$$\dot{x} = Ax + f(t)$$

homogeno: $\dot{x} = Ax$

$$\Phi(t) = e^{At} = P e^{Jt} P^{-1}$$

$$x_n = \Phi(t) x_0$$

nehomogeno:

$$x_p(t) = \int_0^t \Phi(t) \Phi(s)^{-1} \cdot f(s) ds =$$

$$= P e^{Jt} \int_0^t e^{-Js} f(s) ds$$

2 nekonstantu- koeficientu

$$\dot{X} = A(t)X$$

inamo eno rešitev $\begin{bmatrix} x_n \\ y_n \end{bmatrix}$ iščemo drugo rešitev
s komponentama u in v $\begin{bmatrix} u \\ v \end{bmatrix}$

$$\Phi = \begin{bmatrix} x_n & u \\ y_n & v \end{bmatrix} \quad W = \det \Phi = x_n v - u y_n$$

$$W = W(t_0) e^{\int_{t_0}^t \text{tr}(A(s)) ds}$$

$\dot{X} = A(t)X + f(t)$ $\Rightarrow$ variacija konstante

$$\Phi \begin{bmatrix} A' \\ B' \end{bmatrix} = f(t)$$

$$X_p = A \begin{bmatrix} x_n \\ y_n \end{bmatrix} + B \begin{bmatrix} u \\ v \end{bmatrix}$$