

$$3 \quad y:B \vdash \text{const}_y : A \rightarrow B$$

$$\frac{\frac{}{y:B, x:A \vdash y:B} Ax}{y:B \vdash \lambda x:A. y : A \rightarrow B}$$

$$\text{const}_y (f \ z) \xrightarrow{\text{rewrite}} y$$

(preposition and predicate?)

$$(\lambda x:A. t) s \longrightarrow t[x \mapsto s]$$

$$\begin{aligned} \text{const}_y (f : z) &\longrightarrow (\lambda x:A. y) (f \ z) \longrightarrow \\ &\longrightarrow y[x/f \ z] = y \end{aligned}$$

$$f (\text{const}_y \ x) \longrightarrow f (y[x \mapsto x]) \longrightarrow f \ y$$

$$\vdash \quad A+B \longrightarrow B+A$$

$$\frac{\frac{\frac{}{t:A+B \vdash t:A+B} Ax \quad \frac{\frac{}{A \vdash A} Ax \quad \frac{}{B \vdash B} Ax}{A \vdash B+A} \quad \frac{}{B \vdash B+A}}{A+B \vdash B+A} \rightarrow I}{\vdash A+B \longrightarrow B+A} \rightarrow I$$

$$\frac{\frac{\frac{}{t:A+B \vdash t:A+B} Ax \quad \frac{\frac{}{t:A, B \ a:A \vdash A} Ax \quad \frac{}{t:B \vdash B} Ax}{t:A, B \ a:A \vdash in_2(a):B+A} \quad \frac{}{t:B \vdash in_1(b):B+A} I_1}{t:A+B \vdash case \ t \quad \begin{array}{l} | in_1(a) | \rightarrow in_2(a) \\ | in_2(b) | \rightarrow in_1(b) \end{array}} \rightarrow E}{\vdash \lambda t:A+B. case \ t \quad \begin{array}{l} | in_1(a) | \rightarrow in_2(a) \\ | in_2(b) | \rightarrow in_1(b) \end{array} : A+B \longrightarrow B+A} \rightarrow I$$

$$\begin{array}{c}
 \frac{}{f: A \times (B+C) \vdash f_1 A \times (B+C)} \text{Ax} \\
 \hline
 \frac{}{f: A \times (B+C) \vdash \text{snd}(f) B+C} \text{XE}_2 \quad \frac{}{f: A \times (B+C) \vdash b: B} \text{Ax} \quad \frac{}{f: A \times (B+C), c: C \vdash c: C} \text{Ax} \\
 \hline
 f: A \times (B+C) \vdash \text{case } \text{snd}(f) \begin{array}{l} \text{in}_1(b) \mapsto f_1(f), b \\ \text{in}_2(c) \mapsto f_2(f), c \end{array} \quad A \times B + A \times C \\
 \hline
 \vdash \lambda f. \text{case } \text{snd}(f) \begin{array}{l} \text{in}_1(b) \mapsto f_1(f), b \\ \text{in}_2(c) \mapsto f_2(f), c \end{array} A \times (B+C) \rightarrow A \times B + A \times C \rightarrow I
 \end{array}$$

$$\frac{\boxed{\begin{array}{c} x:A \\ \vdots \\ t:B_x \end{array}}}{\lambda x.A, t: \prod_{x:A} B_x} \Pi$$

$$\perp := \prod_{P:\text{Prop}} P$$

$$\frac{f: \perp}{\perp\text{-elim}(f): P} \perp\text{-elim}$$

$$\varphi \vee \psi := \prod_{P:\text{Prop}} . (\varphi \rightarrow P) \rightarrow (\psi \rightarrow P) \rightarrow P$$

$$\frac{t: \varphi \vee \psi \quad \boxed{\begin{array}{c} x:\varphi \\ \vdots \\ s:P \end{array}} \quad \boxed{\begin{array}{c} y:\psi \\ \vdots \\ u:P \end{array}}}{\text{match} \cdot t} \text{match}$$

$$\begin{array}{l} \text{in}_1 x \mapsto s \\ \text{in}_2 y \mapsto u \end{array} : P$$

$$\exists x:A. \varphi x := \prod_{P:\text{Prop}} (\prod_{x:A} (\varphi x \rightarrow P)) \rightarrow P$$

$$\forall x:A. \varphi x := \prod_{x:A} \varphi x$$

$$\varphi \wedge \psi := \prod_{P:\text{Prop}} (\varphi \rightarrow \psi \rightarrow P) \rightarrow P$$

③ let $2 = \{0, 1\}$

$$\underline{\varphi_0 \vee \varphi_1 = \exists x:2. \varphi_x}$$

$$\varphi_0 \vee \varphi_1 := \prod P: \text{Prop}. (\varphi_0 \rightarrow P) \rightarrow (\varphi_1 \rightarrow P) \rightarrow P$$

$$\exists x:2. \varphi_x = \prod P: \text{Prop}. (\prod x:2. \varphi_x \rightarrow P) \rightarrow P =$$

$$= \prod P: \text{Prop}. (\varphi_0 \rightarrow P) \times (\varphi_1 \rightarrow P) \rightarrow P =$$

$$= \prod P: \text{Prop}. (\varphi_0 \rightarrow P) \rightarrow (\varphi_1 \rightarrow P) \rightarrow P$$

$$\textcircled{4} \quad \forall x:A. P_x \rightarrow Q \rightarrow \exists x:A. P_x \rightarrow Q : P:A \rightarrow \text{Prop}, Q:\text{Prop}$$

$$(\prod x:A. P_x \rightarrow Q) \rightarrow (\prod \phi:\text{Prop}. (\prod x:A. P_x \rightarrow \phi) \rightarrow \phi) \Rightarrow Q$$

$$1. \quad t : \prod x:A. P_x \rightarrow Q \quad \text{Assumption}$$

$$2. \quad S : \prod \phi:\text{Prop} (\prod x:A. P_x \rightarrow \phi) \rightarrow \phi \quad \text{Assumption}$$

$$3. \quad Q : \text{Prop}$$

Assumption

$$4. \quad S \, Q : (\prod x:A. P_x \rightarrow Q) \rightarrow Q \quad \text{PIE} \quad (2)(3)$$

$$5. \quad S \, Q \, t : Q$$

$\rightarrow E(4)(1)$

$$6. \quad \lambda t. \lambda S. S \, Q \, t : *$$

$$R_N : \prod_{P:N \rightarrow \text{Type}} (P_0 \rightarrow \prod_{n:N}. P_n \rightarrow P(S_n)) \\ \rightarrow \prod_{n:N}. P_n$$

$$R_N P P_0 P_S 0 = P_0$$

$$R_N P P_0 P_S S_n = P_S n (R_N P P_0 P_S n)$$

$$\text{add} : \mathbb{N} \rightarrow \mathbb{N} \rightarrow \mathbb{N}$$

$$\text{Add} n : \mathbb{N} \rightarrow \mathbb{N}$$

- $P_0 : \text{Add } 0 ; P_0 := \text{id}_{\mathbb{N}}$
- $P_S n : \text{Add } n \rightarrow \text{Add } S_n$
 $f \mapsto (m \mapsto S(f(m)))$

$$\text{add} := R_N \text{Add } P_0 P_S$$

Alternatives

$$\text{add } n : \mathbb{N} \rightarrow \mathbb{N}$$

$$P_k := \text{Add } k := \mathbb{N}$$

$$P_0 = n$$

$$P_S k m :=$$

$$\text{add } n := R_N \text{Add } P_0 P_S$$

define $\text{exp} : \mathbb{N} \rightarrow \mathbb{N} \rightarrow \mathbb{N}$

fix n :

$\text{exp } n : \mathbb{N} \rightarrow \mathbb{N}$

$\mathcal{P}_0^n := S_0$

$\mathcal{P}_s^n \ k \ \underset{n^k}{m} := \text{mul } n \ m$

$\text{exp } n := \mathcal{R} (\lambda _ . \mathbb{N}) \ \mathcal{P}_0^n \ \mathcal{P}_s^n$

① Define $\text{mul} : \mathbb{N} \rightarrow \mathbb{N} \rightarrow \mathbb{N}$

fix $n : \mathbb{N}$ $\text{mul } n : \mathbb{N} \rightarrow \mathbb{N}$

$P_0 := 0$ " $n \cdot k$ "

$P_S k \ m'' = \text{add } m \ n$

$\text{mul } n : \mathbb{R}_{\mathbb{N}} (\lambda _ . \mathbb{N}) \ P_0 \ P_S$

↑ dološi katerega tipa so nji

$m = k \cdot n$ ko že vemo kaj $k \cdot n$ je,
potem rabimo $n(k+1)$

$$\frac{A : \text{Type} \quad x, y : A}{x =_A y : \text{Prop}}$$

$$\frac{x : A}{\text{refl}_A x : x =_A x}$$

$$J := \prod_A \prod P : \prod_{x, y : A} \prod p : x =_A y. \text{Type}.$$

$$(\prod_{x : A} P \ x \ x \ (\text{refl}_A x)) \rightarrow \prod_{x, y : A} \prod p : x =_A y. P \ x \ y \ p$$

$$J_A \ P \ f \ x \ x \ (\text{refl}_A x) \doteq f \ x$$

⑨ Show $\forall x, y : A \ x =_A y \Rightarrow y =_A x$

ne table

10 show

$$\prod_{x,y,z:A} x =_A y \times y =_A z \rightarrow x =_A z$$

↑

$$(x,y,z:A) \rightarrow x =_A y \rightarrow y =_A z \rightarrow x =_A z$$

↑

$$(x,y:A) \rightarrow (z:A) \rightarrow x =_A y \rightarrow y =_A z \rightarrow x =_A z$$

↑

$$(x,y:A) \rightarrow x =_A y \rightarrow (z:A) \rightarrow y =_A z \rightarrow x =_A z$$

↑

$$p \times y \times p : (z:A) \rightarrow y =_A z \rightarrow x =_A z$$

$$f : (x:A) \rightarrow \underbrace{p \times x \text{ refl } x}_{: (z:A) \rightarrow x =_A z \rightarrow x =_A z}$$

$$: (z:A) \rightarrow x =_A z \rightarrow x =_A z$$

$$f_x := \lambda _ . \lambda p . p$$

$\lambda p f \cdot$ je dokaz

$$\frac{A: \text{Type}}{\text{nil} : \text{List } A} \quad \frac{x:A \quad xs:\text{List } A}{\text{cons } x \text{ } xs : \text{List } A}$$

$$R_{\text{List } A} : \prod (P : \text{List } A \rightarrow \text{Type}).$$

$$\begin{aligned} & (P \text{ nil}) \\ & \rightarrow (\prod x:A, xs:\text{List } A. \overset{N}{P \text{ } xs} \rightarrow P \overset{N}{(x::xs)}) \\ & \rightarrow \prod xs:\text{List } A. P \text{ } xs \end{aligned}$$

⑤

$$\text{length} : \text{List } A \rightarrow \mathbb{N}$$

$$P \text{ nil} := \mathbb{N}$$

$$P \text{ } xs := \mathbb{N}$$

$$f := \lambda x:A. \lambda xs:\text{List } A. \lambda m:P \text{ } xs. S m$$

$$P \text{ nil} = 0$$

$$P \text{ cons } m = S m$$

$$\text{length} = R_{\text{List } A} (\lambda_N) \quad 0 \quad f \quad p \text{ cons}$$

$$\text{length} : \text{List } A \rightarrow \mathbb{N}$$

$$P : \text{List } A \rightarrow \text{Type}$$

$$P [] = \mathbb{N}$$

$$f_1 \quad \lambda x:A \quad \lambda xs:\text{List } A. \quad \lambda l:\mathbb{N}. \quad l+1)$$

$$R_{\text{List } A} _ _ 0$$

$$: \prod_{xs:\text{List } A}. \mathbb{N}$$

$$R_{\text{List } A} (\lambda _.\mathbb{N}) \circ (\lambda _:\text{List } A. \lambda _:\mathbb{N}. l+1)$$

$$: \prod_{xs:\text{List } A}. \mathbb{N}$$

const list A $\rightarrow$ List A $\rightarrow$ List A

R_{ListA} ($\lambda_.$ ListA) ...

no table

Vektorji

$$\frac{}{[] : \text{Vec } A \ 0} \qquad \frac{x : A \quad xs : \text{Vec } A \ n \quad n : \mathbb{N}}{x :: xs \text{ Vec } A \ (S n)}$$

$R_{\text{Vec } A} : \prod P : \prod n \in \mathbb{N}. \text{Vec } A \ n \rightarrow \text{Type}.$

$P \circ []$

$\rightarrow (\prod n : \mathbb{N}. \prod x : A. \prod xs : \text{Vec } A \ n. P \ n \ xs \rightarrow P \ (S n) \ (x :: xs))$

$\rightarrow \prod n : \mathbb{N}. \prod xs : \text{Vec } A \ n. P \ n \ xs$

to list $\prod n: \mathbb{N} \text{ vec } A \ n \rightarrow \text{List } A$

$R_{\text{vec } A} (\lambda _, \text{List } A) []_{\text{List } A}$

$(\lambda _: \mathbb{N}. \lambda x: A. \lambda xs: \text{vec } A \ n. \lambda rep: \text{List } A. x :: rep)$